Math Objectives

- Students will explore how the area of a parallelogram remains unchanged if the base and height remain fixed while the side length changes.
- Students will explain a proof of the Pythagorean Theorem and its converse (CCSS).
- Students will construct viable arguments and critique the reasoning of others (CCSS Mathematical Practice).
- Students will reason abstractly and quantitatively (CCSS Mathematical Practice).

Vocabulary

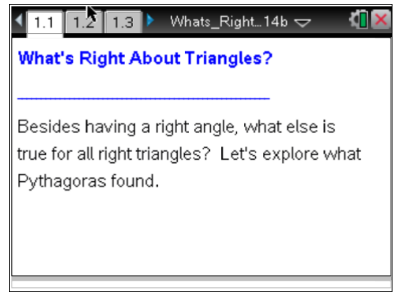
- right triangles
- hypotenuse and legs of a right triangle
- Pythagorean Theorem
- converse of a theorem
- area and perimeter
- parallelogram
- base and height

About the Lesson

- This lesson involves examining a visual proof of the Pythagorean Theorem and supporting what happens geometrically.
- As a result, students will:
  - Explore the invariability of the areas of parallelograms that keep their base and height fixed but have different side lengths. This understanding will help students support what is happening with the visual proof of the Pythagorean Theorem.
  - Examine what kind of triangles have the sum of the squares of their shorter sides equal to the square of their largest side in order to explore the converse of the theorem.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software



Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:
Student Activity

- What's_Right_About_Triangles_Student.pdf
- What's_Right_About_Triangles_Student.doc

TI-Nspire document

- Whats_Right_About_Triangles.tns

Discussion Points and Possible Answers

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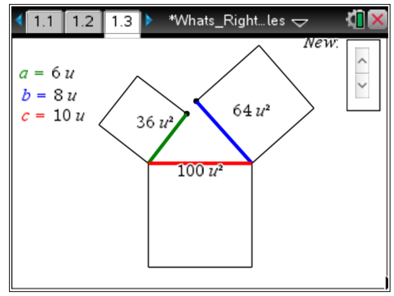

The Pythagorean Theorem describes a relationship between the **areas of squares** built off of each of the 3 sides of right triangles (see figure to the right). You will explore what is true about the sums of these areas for different triangles by dragging sides of triangles (with squares built off of these sides) to see what happens and to see if the Pythagorean Theorem holds true for non-right triangles as well (see Page 1.3 to the right).

Teacher Tip: The rule for the Pythagorean Theorem is not directly stated here to allow the students to "uncover" the relationship again for themselves through the activity. If your students have not heard of the Pythagorean Theorem in earlier studies, you may still choose to allow the students to find the relationship by examining the sums they find in the following table. There are 10 different triangles in the exploration that are generated "randomly" as the students press on the New slider. The ten cases are completed in the table below.

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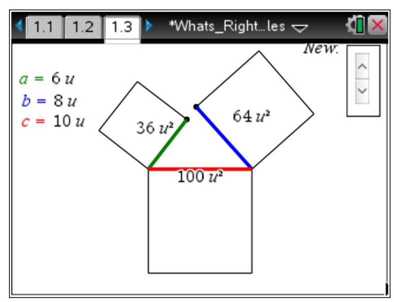

1. Select the closed circles to see if you can make a triangle with the given side lengths. Record your findings in the table below. If you cannot connect the sides to form a triangle, you should leave the "measure" column blank and write "no" in the "right triangle" column. Complete several examples.

Sample Answers:

a^2	b^2	c^2	Measure of new angle formed	Right triangle? (Yes or No)
$36u^2$	$64u^2$	$100u^2$	90º	Yes
$9u^2$	$25u^2$	$36u^2$	94º	No
$9u^2$	$9u^2$	$64u^2$		No
$9u^2$	$16u^2$	$25u^2$	90º	Yes
$25u^2$	$144u^2$	$169u^2$	90º	Yes
$16u^2$	$16u^2$	$16u^2$	60º	No
$16u^2$	$16u^2$	$49u^2$	122º	No
$16u^2$	$36u^2$	$100u^2$		No
$25u^2$	$25u^2$	$49u^2$	89º	No
$81u^2$	$144u^2$	$225u^2$	90º	Yes

TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

2. a. Based upon your completed table, what is true about the sums of the areas of the squares for right triangles?

Sample Answer: The sum of the areas of the squares off of the two legs is equal to the area of the square off of the longest side (hypotenuse) or $a^2 + b^2 = c^2$.

b. Does this relationship hold for non-right triangles?

Sample Answer: No, the rule does not work for non-right triangles.

c. So if you know that $a^2 + b^2 = c^2$, for the sums of squares built off of the 3 sides of a triangle, then what kind of triangle do you have to have?

Sample Answer: If $a^2 + b^2 = c^2$, then I have a right triangle.

Teacher Tip: At this point, students are actually investigating the converse of the Pythagorean Theorem. Students will see from the table of collected information that when the sum of the areas off of the two legs of a triangle is equal to the area of the square built off of the longest side, they have a right triangle. This idea, if $a^2 + b^2 = c^2$, then I have a right triangle, represents the converse of the Pythagorean Theorem, i.e. for right triangles, $a^2 + b^2 = c^2$.
This activity also allows the students to note that given any 3 side-lengths, you cannot always make a triangle. You could return to this activity when exploring the Triangle Inequality and finding that the sum of any two sides of a triangle must be larger than the third. At this point, the table is for finding a relationship with the areas of squares built off of the triangle sides.

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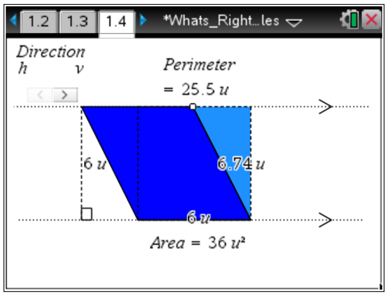


Let the base be the bottom side of the parallelogram. The height is the perpendicular distance between the base and its opposite side.

3. Horizontally drag the open circle on the parallelogram. Describe what is happening to the parallelogram's
   a. base:

      <u>Sample Answer:</u> The base keeps the same measure.

   b. height:

      <u>Sample Answer:</u> The height remains the same measure.

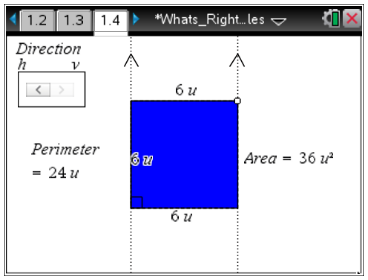
   c. non-horizontal side lengths:

      <u>Sample Answer:</u> The side lengths change measure as I drag the parallelogram.

4. With the above information, explain why the perimeter of the parallelogram is changing but the area is not.

   <u>Sample Answers:</u> The perimeter for a parallelogram is the sum of the 4 side lengths. The perimeter is changing since two of the side lengths are changing as I drag the parallelogram. This makes the sum of the 4 sides change. The area stays the same since the area of a parallelogram is A=*bh*; the base and the height are not changing as I drag the parallelogram.

5. Change the sliding direction of the open circle to vertical (by selecting *v* in the upper left corner), and see what happens to the perimeter and area as you drag the open circle on the parallelogram. **Describe** what you find and **explain** why it is happening.



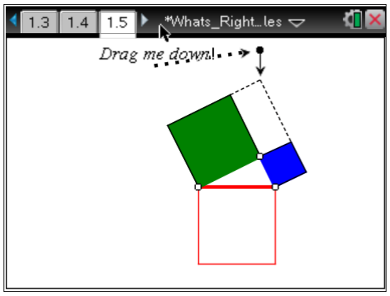
Sample Answers: The perimeter does change again since two sides are changing length as I drag the parallelogram. The area stays the same since the base and height do not change; area of a parallelogram is *A=bh*.

You will use what you have found here with parallelograms to help explain what is happening next with a visual proof of the Pythagorean Theorem.

TI-Nspire Navigator Opportunity: *Live Presenter*
See Note 2 at the end of this lesson.

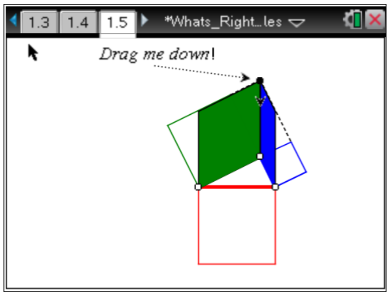
Teacher Tip: Allow time for the students to understand what they see happening visually and then to mathematically justify why it is true before moving onto the next question. Too often students are caught up in the visual change of the shape and believe that all aspects of the shape must be changing and don't believe the area can remain fixed.

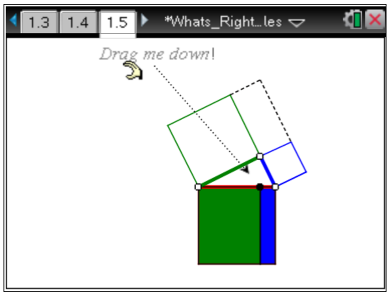
Move to page 1.5.



6. On this page, you should see squares built off of a right triangle's sides. Drag the closed circle downward, and describe what happens with the areas of the two squares.

Sample Answers: The area of the two squares on the legs of the right triangle fill up the area of the larger square on the hypotenuse.





7. Using what you know about the area of a parallelogram where the base and height don't change, explain how the moving shapes prove the Pythagorean Theorem for right triangles.

Sample Answers: As I started dragging the point, I changed the appearance of the squares. When the dragging point reached the spot where the two squares sides met, I had made two shapes that were now parallelograms but they had the same areas as the original squares. When I continued to drag the point downward, the parallelograms kept their same shape until they reached the bottom side of the square built off of the hypotenuse. At that point, the parallelograms started to change shape again; but since their height and base stayed the same, their areas didn't change. Finally, both parallelograms became rectangles as I continued to drag the point down. The areas of the rectangles filled up the area of the square. Since the original rectangular areas were from the two squares built off of the legs of the right triangle, they represent the quantity $a^2 + b^2$. The area of the square off of the hypotenuse is c^2 and since the other two areas fill the area of that square, that shows that $a^2 + b^2 = c^2$.

Teacher Tip: It is not mentioned on the student worksheet, but students can also change the size of the square off of the hypotenuse to see that the two areas from the legs of the right triangle will fill the area of the 3rd square for many different sizes of "c^2".

TI-Nspire Navigator Opportunity: ***Live Presenter***
See Note 3 at the end of this lesson.

Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- How to describe/explain the Pythagorean Theorem and give examples of when it applies.
- How to explain a visual proof of the Pythagorean Theorem.
- How to describe/explain the converse of the Pythagorean Theorem.
- How to explain how a parallelogram's area could remain fixed but the perimeter change.

Assessment

TI-Nspire Navigator Opportunity: ***Quick Poll***
See Note 4 at the end of this lesson.

Which of the following side lengths of triangles would make a right triangle?

a. a = 4u, b = 5u, c = 6u
b. a = 8u, b = 8u, c = 16u
c. a = 12u, b = 16u, c = 20u
d. a = 5u, b = 5u, c = 5u

Sample Answers:

c. a = 12u, b = 16u, c = 20u

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Note 1
Question 1, Quick Poll
You can use Quick Poll to see which triangles students are finding to be right triangles. You could ask students to provide one set of 3 numbers they have found to make a right triangle. A variety of responses could appear, prompting a discussion of any relationship with the sum of the areas.

Note 2
Question 5, Live Presenter
Have students share their explanation of why the perimeter changes but the area remains fixed.

Note 3
Question 7, Live Presenter
Have some students share their explanation of the visual proof for the Pythagorean Theorem, allowing other students to comment and ask questions.

Note 4
Assessment, Quick Poll
You can use Quick Poll to see if students can apply the converse of the Pythagorean Theorem.

> **Tech Tip:** There is a Quick Poll option "Allow document access" under the Tools menu. You might decide to have that turned on or off, depending upon whether you want the students to be able to return to the TI-Nspire document or not while they answer the Quick Poll.