Teacher Notes

Activity 6

Local Linearity, Differentiability, and Limits of Difference Quotients

Objectives

- Connect the concept of local linearity to differentiability through numerical explorations of limits of difference quotients
- Develop a flexible, coordinated, multi-representational image of the limiting process as it relates to average rates of change passing to instantaneous rates of change

Materials

- TI-84 Plus / TI-83 Plus

Teaching Time

- 100 minutes

Abstract

In this activity, the local linearity of several functions at different points is discussed and explored. The concept of local linearity is linked to differentiability by examining one-sided limits of difference quotients of several functions numerically as *h* approaches zero. By the end of the activity, students will see examples of finite, one-sided difference quotient limits that are not equal, and one-sided difference quotient limits that approach infinity. These behaviors (as well as difference quotient limits that exist) are linked to the graphical behavior of the function.
This activity contains 20 questions and 2 extension exercises. Most questions require students to interpret the information displayed on the graphing handheld using the result of a graphical or numerical exploration. A few questions may require some algebraic manipulation.

Management Tips and Hints

Prerequisites

- Students should be able to graph and modify (zoom, trace, and so on) graphs of functions and generate tables using the graphing handheld.
- This activity is appropriate after the derivative and differentiability are defined.

Evidence of Learning

Students can investigate the local linearity given a function and a point and then connect that notion with the function's differentiability at that point.

Common Student Errors/Misconceptions

- The difference quotient uses two literal symbols, x and h. Moreover, the value of x is often fixed at some arbitrary real number often symbolized as a. Students often confuse the roles of these symbols.
- It is necessary to use T in place of h when exploring on the graphing handheld. Questioning students about the roles and values of x and h as they work through the activity may help students examine the roles more closely.

Extensions

1. Driver Dan claims that during a one-hour trip his displacement function is $s(t) = |t - 30|$ where t is measured in minutes and s is measured in miles. Is this possible? Explain.

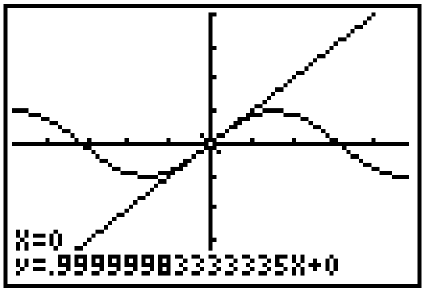
2. For Driver Dan's alleged trip:

a. Draw the displacement versus time graph.

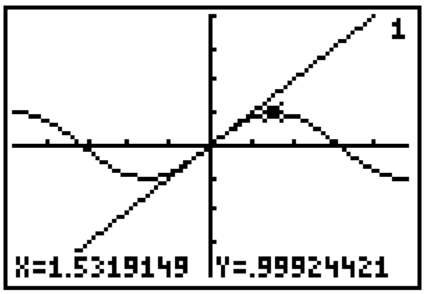
b. Draw the velocity versus time graph.

Activity Solutions

1. No; it does not show the sine function's wider properties. The window is so small that the graph looks like a line.
2. y = 0.99999983333335x + 0, which is very close to $y = x$.
3. Yes



4. (1.5319149, 0.99924421)



5. Vertical distance at x = 1.5319149
   - To line: (0.99999983333335) (1.5319149) = 1.531914645
   - To curve: sin (1.5319149) = 0.9992442125
   - Difference: 0.5326706875
6. Points will vary, but $f(x) = \sin(x)$ should appear locally linear everywhere.
7. No; no amount of zooming in will get rid of the sharp corner.
8. No; there is a cusp at (1, 0).
9. Yes; the difference quotient appears to approach 1 as T approaches 0 from the right. Students should look from the top of the table down toward T = 0.

T	X_{1T}	Y_{1T}
.3	0	.98507
.2	0	.99335
.1	0	.99833
0	0	ERROR
-.1	0	.99833
-.2	0	.99335
-.3	0	.98507

T=.3

10. Yes; the difference quotient appears to approach 1 as T approaches 0 from the left. Students should look from the bottom of the table up toward $T = 0$.

11. Yes

12. The difference quotient is getting larger. It approaches infinity as T approaches 0 from the right.

T	X_{1T}	Y_{1T}
.03	1	3.2183
.02	1	3.684
.01	1	4.6416
0	1	ERROR
-.01	1	-4.642
-.02	1	-3.684
-.03	1	-3.218

T=.03

13. The difference quotient is getting smaller. It approaches negative infinity as T approaches 0 from the left.

14. The limit does not exist because both one-sided limits are not finite and not equal to each other.

This means that $f(x) = \sqrt[3]{(x-1)^2}$ is not differentiable at $x = 1$ or that f is not locally linear at (1, 0).

15. The difference quotient approaches 2 as T approaches 0 from the right.

T	X_{1T}	Y_{1T}
.03	1	2.03
.02	1	2.02
.01	1	2.01
0	1	ERROR
-.01	1	1.99
-.02	1	1.98
-.03	1	1.97

T=.03

16. The difference quotient approaches 2 as T approaches 0 from the left.

17. The one-sided limits are equal to each other, so the limit may exist.

18. Yes; the difference quotient is 1 for all positive T.

T	X_{1T}	Y_{1T}
.03	2	1
.02	2	1
.01	2	1
0	2	ERROR
-.01	2	-1
-.02	2	-1
-.03	2	-1

T=.03

19. Yes; the difference quotient is $^-1$ for all negative T.

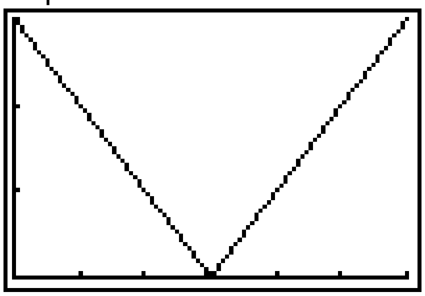
20. No; the one-sided limits are not equal to each other.

Extension Solutions

1. Dan's trip is impossible because his displacement function would dictate an instantaneous velocity change from ⁻1 miles per minute to +1 miles per minute.

2.

   a. Displacement versus Time



   b. Velocity versus Time