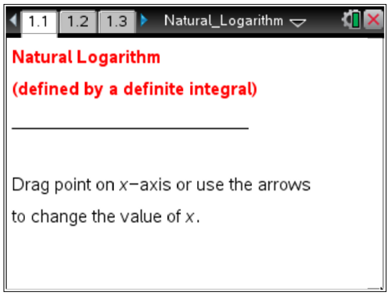
Natural Logarithm

Student Activity

Name ________________

Class ________________

Open the TI-Nspire document *Natural_Logarithm.tns*.



The purpose of this activity is to introduce one definition of the natural logarithm function, that is, $\ln x = \int_1^x \frac{1}{t}dt$. This activity allows you to visualize this definition and to discover some of the properties of the natural logarithm function and its graph.

One way to define the natural logarithm function and to develop properties of this function involves the area under the graph of $y = \frac{1}{x}$.

To begin this activity, let the natural logarithm function, denoted ln, be defined by $\ln x = \int_1^x \frac{1}{t}dt$, for $0 < x < \infty$. (We'll see why there are some restrictions on the domain.) An interpretation of this definite integral is the area under the graph of $y = \frac{1}{x}$, above the x-axis, and between the vertical lines at 1 and x. Using this geometric interpretation of the definite integral, you will learn some of the characteristics of the graph of $y = \ln x$ and properties of the natural logarithm function.

Move to page 1.2.

1. As you grab and drag point x to the right along the horizontal axis or use the up and down arrows the top-right portion of the page, the computed area of the shaded region is equivalent to $\ln x$, the value of the natural logarithm function.

> **Tech Tip:** To easily change the value of x, select the up and down in the top-right portion of the page. Also you can select the value and type in the number.
>
> **Tech Tip:** With the iPad, touch your finger to the point and then drag it along the x-axis.

a. Complete the following table.

x	1	1.5	2	4	6	8
$\ln x$						

b. Explain what happens to the value of $\ln x$ as x increases.

c. Explain your answer in part b geometrically.

2. Drag point x to the left of 1 (but greater than 0), or use the up and down arrows to change the value of x.

a. Complete the following table.

x	1	0.9	0.7	0.5	0.2	0.1	0.05
$\ln x$							

b. Explain what happens to the value of $\ln x$ as x decreases (gets closer to 0).

c. Explain your answer in part b geometrically.

Move to page 1.3.

3. A part of the graph of $y = \ln x$ is displayed. Grab point x or use the up and down arrows to change the value and move it along the horizontal axis to the right to construct the remaining part of the graph of $y = \ln x$. The values of the natural logarithm function are displayed on the right screen.

a. Explain what happens to the graph of $y = \ln x$ as x increases without bound (as $x \to \infty$).

b. Explain what happens to the graph of $y = \ln x$ as x approaches 0 from the right (as $x \to 0^+$).

c. Explain why $x = 0$ is not in the domain of the function $y = \ln x$.

d. The function $\mathbf{f}(x) = \frac{1}{x}$ is defined for $x < 0$. For example, $\mathbf{f}(-2) = -\frac{1}{2}$. Explain why the definition of the natural logarithm function cannot be extended to include negative numbers.

e. Use the Fundamental Theorem of Calculus to find the derivative of $\mathbf{f}(x) = \ln x$. Determine the intervals on which the graph of $y = \mathbf{f}(x)$ is increasing and the intervals on which it is decreasing. Find the absolute extreme values for **f.** Determine the intervals on which the graph of $y = \ln x$ is concave up and the intervals on which it is concave down.