

MATH NSPIRED

Background

The purpose of this file is to enable users to numerically investigate the limit of a function as $x \to a^+$ and as $x \to a^-$, or the limit as $x \to \infty$ and as $x \to -\infty$. Sliders are used to quickly examine the behavior of each function, and a table of values is also computed and displayed.

Course and Exam Description Unit

1.4: Estimating Limit Values from Tables1.15: Connecting Limits at Infinity ad Horizontal Asymptotes

Calculator Files

LimitSliders.tns

Using the Document

LimitSliders.tns: This calculator file provides a technology tool for investigating the limit of an arbitrary function as x approaches a specific value from the left and from the right. In a separate calculator problem, the user can investigate the limit of a function as x increases or decreases without bound, that is, as $x \to \infty$ or as $x \to -\infty$. A short table of values for each type of limit is also automatically computed and displayed in a Lists and Spreadsheet page.

The default limits are

$\lim_{x \to 3} \frac{x^2 - 9}{3 - x}$	(ca	alculator problem 2)	
and			
$\lim_{x \to \infty} \tan^{-1} x$	and	$\lim_{x \to -\infty} \tan^{-1} x$	(calculator problem 3)

Page 1.1

- 0 -	
 ▲ 1.1 2.1 2.2 LimitSliders ▶ LimitSliders ▶ Sliders for investigating limits of functions Page 2.1: a slider that substitutes into f (x) x-values that approach the real number a (from both the left and the right sides of a) 	This introductory screen provides information to help utilize this tns file. In examining either the limit as x approaches a specific number, or as xincreases or decreases without bound, the function is defined as $f(x)$. A slider is used to change the value of x .
Page 3.1: a slider that substitutes into $f(x)$ x-values that "approach" ∞ and $-\infty$ (grow large in magnitude through + and - values) [Tables of values are on Pages 2.2 and 3.2]	A table of values is given on pages 2.2 and 3.2.

Exploring Limits of Functions

TEACHER NOTES

Page 2.1

I.1 2.1 2.2 ▶ LimitSliders RAD X X	Page 2.1 is a Notes page that contains two math boxes,
× ² 0	values of the function f , and a slider to change the value
Define $f(x) = \frac{x - 9}{2} + Done$ a:=3 + 3	of <i>x</i> . The function is defined in the first math box, and the
$\lim_{x \to \infty} \frac{3-x}{f(4)} = -7$	value of a is set in the second math box. The function f is
$x \rightarrow a^+$	initially evaluated at $a \pm 1$. As you click the slider, the value
$\lim_{x \to a} (f(x)) = -5$	of x move closer to a from the right and from the left in
$ \begin{array}{c} \text{unif} (\mathbf{u}(\mathbf{x})) & \mathbf{v}(2, \mathbf{x}) = -5, \\ \mathbf{x} \rightarrow \mathbf{a}^{-1} \end{array} $	steps of 10^{-n} , for $n = 1, 2, 3, 4, 5$. This provides data that can
	be used to guess the overall limit, as indicated in the lower
$x \rightarrow a \ slider \qquad \qquad$	right side of the screen.
<i>x</i> → a	
	Note that there are calculations at the bottom of this
	screen, used to compute and display the values of x and
	the function values.

Page 2.2

<u> </u>					
1.1	1.1 2.1 2.2 *LimitSliders RAD 🗍 🗙				This Lists and Spreadsheet page contains the values of x
/	A xleft	в	C xright	D	close to <i>a</i> on the left and the right, and the corresponding
=		=f(xleft)		=f(xright)	values of the function f . This spreadsheet presentation
1	2.	-5.	4.	-7.	might make it easier to conjecture if the values of f are
2	2.9	-5.9	3.1	-6.1	converging of diverging.
3	2.99	-5.99	3.01	-6.01	
4	2.999	-5.999	3.001	-6.001	
5	2.9999	-5.9999	3.0001	-6.0001 🗸	
A10			1	4 1	



	4 2.1 2.2 3.1 ▶ *LimitSliders RAD	The function f is defined at the top of this Notes page in
	Define $\mathbf{f}(x) = \tan^{-1}(x) \cdot Done$	the Math Box. By default, the value of the function is given
		for $x = 10$ and $x = -10$. The slider at the bottom of the
·· (-())		page is used to increase the magnitude of each of these x-
	IIII (I(x)) = 1.4/112/0/4	values, up to $\pm 10^7$. The value of the function is computed
	$x \rightarrow \omega$	and displayed automatically. These numerical values for f
	$\lim_{t \to \infty} (\mathbf{I}(\mathbf{x})) \mathbf{r}(-10.) = -1.4/112/0/4$	may provide insight into, or confirmation of, the limits
	$x \rightarrow -\infty$	$\lim_{x \to \infty} f(x)$ and $\lim_{x \to \infty} f(x)$.
	$x \rightarrow \pm \infty$ slider	$x \rightarrow \infty$ $x \rightarrow -\infty$



Page 3.2

1 2	2.2 3.1	3.2	2 ▶ *LimitS	iliders	rad 📋 🗙	This Lists and Spreadsheet page displays the values of x
	А хро	s	в	⊂ xneg	D	used on Page 3.1, and the corresponding values of the
=			=f(xpos)		=f(xneg)	function f . This spreadsheet presentation might make it
1		10.	1.47113	-10.	-1.47113	easier to conjecture if the values of f are converging or
2	1	00.	1.5608	-100.	-1.5608	diverging as x increases or decreases without bound.
3	10	00.	1.5698	-1000.	-1.5698	
4	100	00.	1.5707	-10000.	-1.5707	
5	1000	00.	1.57079	-100000.	-1.57079 🗸	
A7					4 >	

Suggested Applications and Extensions

Use Pages 2.1 and 2.2 to estimate the value of the limit, if it exists.

- 1. $\lim_{x \to 0} \frac{\sin x}{x}$
- $2. \quad \lim_{x \to 1} \frac{\ln x}{x 1}$
- 3. $\lim_{h \to 0} \frac{(2+h)^4 16}{h}$
- $4. \quad \lim_{x \to 0} \frac{x}{e^{2x} 1}$
- 5. $\lim_{x \to 0} \frac{\tan(2x)}{\sin(5x)}$
- $6. \quad \lim_{x \to 0} |x|^x$
- 7. $\lim_{x \to 4} [\sin x + \ln |x 4|]$
- 8. $\lim_{x \to 0} x^2 \frac{2^x}{10000}$
- 9. $\lim_{x \to 0} \sin \frac{1}{x}$
- 10. $\lim_{x \to 0} \frac{1}{1 + e^{1/x}}$
- 11. $\lim_{x \to 0} f(x)$ where $f(x) = \begin{cases} -(x^2 + 1) & x < 0 \\ e^x & x \ge 0 \end{cases}$

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Use Pages 3.1 and 3.2 to estimate the value of the limit, if it exists.

1.
$$\lim_{x \to \infty} \frac{3x^2 + 7x - 11}{15 - 2x - 9x^2}$$

2.
$$\lim_{x \to -\infty} \sqrt{\frac{7x^2 - x - 1}{11 - x + 5x^2}}$$

3.
$$\lim_{x \to \infty} \frac{\sqrt{x} + x}{5x - 3}$$

4.
$$\lim_{x \to \infty} \left(\sqrt{3x^2 + 5} - x\right) \text{ and } \lim_{x \to -\infty} \left(\sqrt{3x^2 + 5} - x\right)$$

5.
$$\lim_{x \to \infty} \frac{x^3}{3^x} \text{ and } \lim_{x \to -\infty} \frac{x^3}{3^x}$$

6.
$$\lim_{x \to \infty} \frac{\ln|x|}{x} \text{ and } \lim_{x \to -\infty} \frac{\ln|x|}{x}$$

7.
$$\lim_{x \to \infty} \frac{\sqrt{3x^2 + 7}}{2x + 5} \text{ and } \lim_{x \to -\infty} \frac{\sqrt{3x^2 + 7}}{2x + 5}$$

8.
$$\lim_{x \to \infty} \frac{\sin x}{e^x} \text{ and } \lim_{x \to -\infty} \frac{\sin x}{e^x}$$

9.
$$\lim_{x \to \infty} \left(\sqrt{x^2 + 3x + 7} - \sqrt{3x^2 - 7x - 5}\right)$$

10.
$$\lim_{x \to \infty} \frac{e^{-x}}{2x + 5} \text{ and } \lim_{x \to -\infty} \frac{e^{-x}}{e^{x}}$$

10.
$$\lim_{x \to \infty} \frac{e}{\tan^{-1} x}$$
 and $\lim_{x \to -\infty} \frac{e}{\tan^{-1} x}$