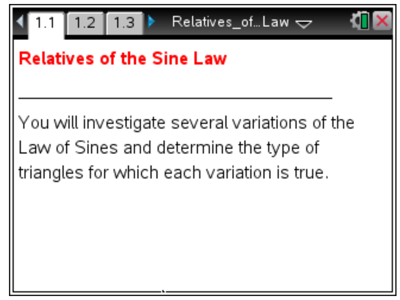
Relatives of the Sine Law

MATH NSPIRED



Math Objectives

- Students will make conjectures about the types of triangles for which certain variations of the Law of Sines are true.
- Students will verify conjectures using algebraic and trigonometric methods.
- Students will use appropriate tools strategically (CCSS Mathematical Practice).
- Students will reason abstractly and quantitatively (CCSS Mathematical Practice).
- Students will construct viable arguments and critique the reasoning of others (CCSS Mathematical Practice).

Vocabulary

- Law of Sines
- Law of Cosines

About the Lesson

- This lesson involves investigating several variations of the Law of Sines.
- As a result, students will:
  - Make conjectures about the type of triangles for which variations of the Law of Sines are true.
  - Verify conjectures using algebra, the Law of Sines, the Law of Cosines, and other trigonometric identities.

TI-Nspire™ Navigator™

- Transfer a .tns File.
- Use Quick Poll to assess students' understanding.

Activity Materials

- Compatible TI Technologies: TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Tech Tips:

- This activity includes screen captures taken from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:

Student Activity

- Relatives_of_the_Sine_Law_Student.pdf
- Relatives_of_the_Sine_Law_Student.doc

TI-Nspire document

- Relatives_of_the_Sine_Law.tns

Discussion Points and Possible Answers

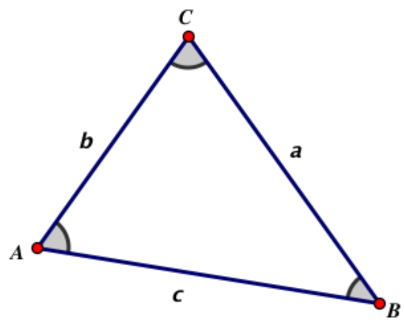
Tech Tip: This document has settings of degrees. It will not work correctly if a student changes the settings to something other than degrees.

Teacher Tips: This activity offers a chance to discuss a useful technique of problem posing- varying the conditions of the problem/theorem – in this case, the Law of Sines. In addition, you can emphasize that using the geometric tools can suggest that a conjecture is true, but deductive methods are needed to verify the truth of any conjecture.

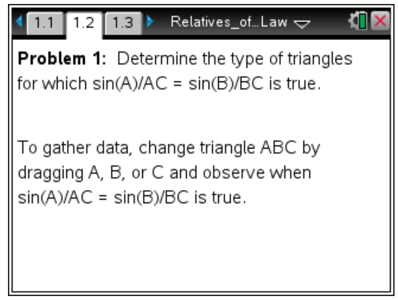
The form $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$ might be easier to remember and use in verifications of conjectures while the form $\frac{\sin A}{BC}=\frac{\sin B}{AC}=\frac{\sin C}{AB}$ should be used for the calculator.

As students gather data, remind them to move all three vertices so that they can find cases on the grid when the desired quantities are equal. For example, finding cases when AC = BC to show an isosceles triangle or C = 90° to show a right triangle.

The Law of Sines states: $\frac{\sin A}{BC}=\frac{\sin B}{AC}=\frac{\sin C}{AB}$ or $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$ for any triangle $\triangle ABC$ with angles A, B, and C and sides $AB = c$, $AC = b$, and $BC = a$. In this activity, you will examine several variations of the Law of Sines and determine the type(s) of triangles for which each variation is true.



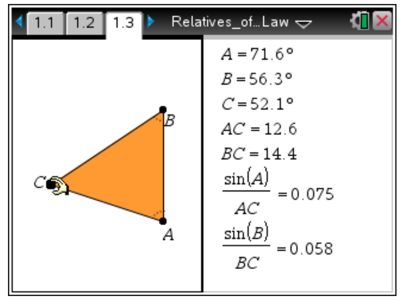
Read page 1.2, and move to page 1.3.



1. Consider $\frac{\sin A}{AC} = \frac{\sin B}{BC}$ $\left(\frac{\sin A}{b} = \frac{\sin B}{a}\right)$. Drag vertices *A*, *B*, and *C* to gather data, and then complete the conjecture below:

   If $\frac{\sin A}{AC} = \frac{\sin B}{BC}$, then $\triangle ABC$ is

   ____.

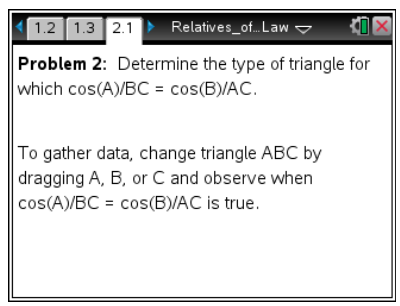


   <u>Answer:</u> $\triangle ABC$ is isosceles. This conjecture is plausible since the two expressions are equal when $\angle A = \angle B$ and $AC = BC$. (Hint: students can more easily discover this if they only move point *C*.)

2. Verify your conjecture using algebra, the Law of Sines, the Law of Cosines, or other "trig identities."

<u>Sample Answers:</u> By the Law of Sines, $\sin B = \frac{b \sin A}{a}$ so that $\frac{\sin A}{b} = \frac{\sin B}{a} \rightarrow \frac{\sin A}{b} = \frac{b \sin A}{a^2}$ $\rightarrow a^2 = b^2 \rightarrow a = b$ since both *a* and *b* are positive numbers. Thus $\triangle ABC$ is isosceles.

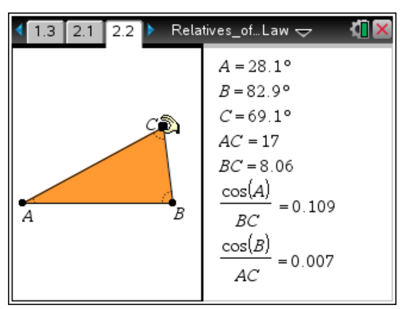
Read page 2.1, and move to page 2.2.



3. Consider $\frac{\cos A}{BC} = \frac{\cos B}{AC}$ $\left(\frac{\cos A}{a} = \frac{\cos B}{b}\right)$. Drag vertices *A*, *B*, and *C* to gather data, and then complete the conjecture below:

   If $\frac{\cos A}{BC} = \frac{\cos B}{AC}$, then $\triangle ABC$ is

   ____.



   <u>Answer:</u> $\triangle ABC$ is isosceles. This conjecture is plausible since the two expressions are equal when $\angle A = \angle B$ and $BC = AC$.

4. Verify your conjecture using algebra, the Law of Sines, the Law of Cosines, or other "trig identities."

Sample Answers:

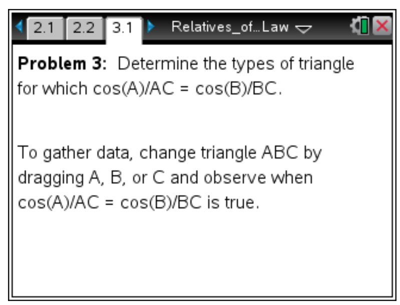
Solution 1: $\frac{\cos A}{a} = \frac{\cos B}{b} \rightarrow \frac{\cos^2 A}{a^2} = \frac{\cos^2 B}{b^2} \rightarrow \frac{1-\sin^2 A}{a^2} = \frac{1-\sin^2 B}{b^2} \rightarrow$ $\frac{1}{a^2} - \frac{\sin^2 A}{a^2} = \frac{1}{b^2} - \frac{\sin^2 B}{b^2} \rightarrow \frac{1}{a^2} = \frac{1}{b^2}$ by the Law of Sines. Thus, $a^2 = b^2$ or $a = b$ since both a and b are positive numbers so that $\triangle ABC$ is isosceles.

Solution 2: By the Law of Cosines, $\frac{\cos A}{a} = \frac{\cos B}{b} \rightarrow \frac{b^2+c^2-a^2}{2abc} = \frac{a^2+c^2-b^2}{2abc} \rightarrow 2b^2 = 2a^2 \rightarrow$ $a = b$ since both a and b are positive numbers. Thus $\triangle ABC$ is isosceles.

Teacher Tip: Ask students to share their answers to see if more than one method of proof can be found.

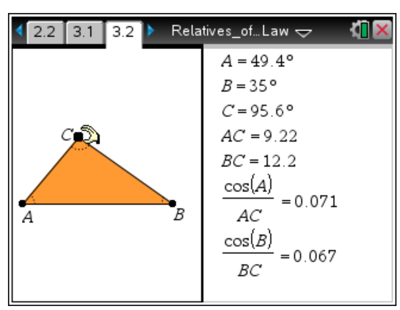
TI-Nspire Navigator Opportunity: *Quick Poll*
See Note 1 at the end of this lesson.

Read page 3.1, and move to page 3.2.



5. Consider $\frac{\cos A}{AC} = \frac{\cos B}{BC}$ $\left(\frac{\cos A}{b} = \frac{\cos B}{a}\right)$. Drag the points *A*, *B*, and *C* to gather data, and then complete the conjecture below:

If $\frac{\cos A}{AC} = \frac{\cos B}{BC}$, then $\triangle ABC$ is _________ or _________.



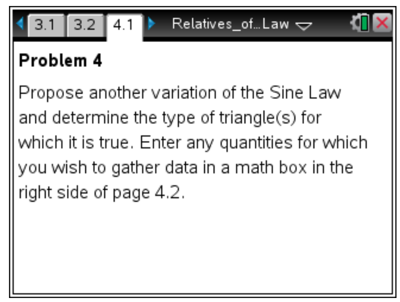
Answer: $\triangle ABC$ *is* isosceles or right. This conjecture is plausible since the two expressions are equal when $\angle A = \angle B$ and $AC = BC$ [isosceles] and $\cos B = \sin A$ and $\cos A = \sin B$ in a right triangle, so that $\frac{\cos A}{AC} = \frac{\cos B}{BC}$ is equivalent to the Sine Law.

6. Verify your conjecture using algebra, the Law of Sines, the Law of Cosines, or other "trig identities."

Sample Answers: By the Law of Cosines,
$\frac{\cos A}{b} = \frac{\cos B}{a} \rightarrow \frac{b^2 + c^2 - a^2}{2b^2c} = \frac{a^2 + c^2 - b^2}{2a^2c} \rightarrow a^2(b^2 + c^2 - a^2) = b^2(a^2 + c^2 - b^2) \rightarrow$
$a^2c^2 - a^4 = b^2c^2 - b^4$ or $c^2(a^2 - b^2) = (a^4 - b^4) = (a^2 + b^2)(a^2 - b^2)$. Thus, either $a^2 = b^2$ [$a = b$] or $c^2 = a^2 + b^2$, that is, $\triangle ABC$ is either isosceles or right.

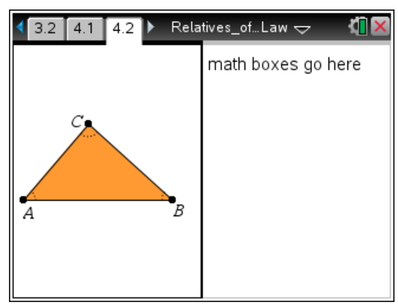
Teacher Tip: Since posing a problem is more difficult than making and proving a conjecture, you might want to assign this next problem in small groups, as homework, or even omit it entirely.

Read page 4.1, and move to page 4.2.



7. Propose another variation (relative) of the Sine Law and then investigate the (type(s) of) triangles for which your variation is true.
   Hint: Consider a variation involving both sine and cosine, or one involving the tangents of the angles of the triangle.

Sample Answers:
$\frac{\sin A}{a} = \frac{\cos B}{b}$ [isosceles right triangles] ;
$\frac{\sin A}{b} = \frac{\cos B}{a}$ [isosceles right triangles]; and
$\frac{\tan A}{a} = \frac{\tan B}{b}$ [isosceles triangles] are three possibilities.



Wrap Up

Upon completion of the lesson, the teacher should ensure that students are able to understand:

- How to gather data about a triangle using geometric tools.
- How to make conjectures based on the data.
- How to verify conjectures using algebraic and trigonometric methods.

TI-Nspire™ Navigator™

Note 1

Name of Feature: Quick Poll

Use a Quick Poll and subsequent discussion to assess students' understanding of various arguments justifying that the conjecture is true, which argument they prefer, and why.