ACTIVITY 12

Bouncing Ball

Math Objectives:

- Graph scatter plots
- Graph and interpret a quadratic function
- Apply the vertex form of a quadratic equation
- Calculate the maximum value of a parabola

Materials:

- TI-83/TI-84 Plus Family
- Calculator-Based Ranger™ (CBR 2™)
- Vernier EasyData™ Application
- Bouncing ball

OVERVIEW

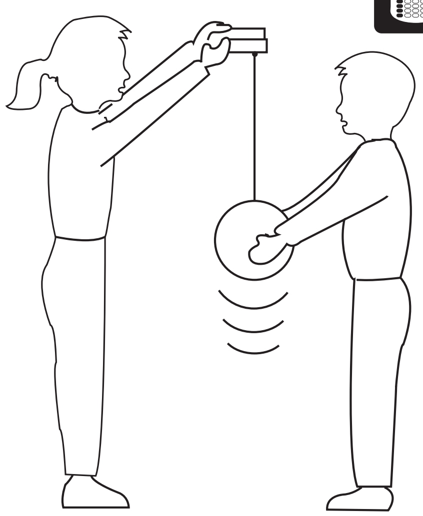
The next activity, **Activity 13: How High Will It Bounce?**, uses the data collected in this activity for investigations into the rebound height of a bouncing ball. Unless you have an extended lab period or block scheduling, it is unlikely you could complete them both in an average class period. Although this lesson can be done alone, if you plan to do Activity 13, be sure to have your students pay special attention to the directions about saving the data into named lists for easy retrieval.

Real-world concepts such as free-falling and bouncing objects, gravity, and constant acceleration are examples of parabolic functions. This activity investigates the values of height, time, and the coefficient *A* in the quadratic equation, $Y = A(X - H)^2 + K$, which describes the behavior of a bouncing ball.

NOTE *For help with saving data to named lists,* *see Appendix E.*

NOTE Demo the activity using the overhead calculator so the entire class can see the process. If you only have one CBR 2, after running the activity link the data lists to each student's calculator. If you have enough CBR 2 units, have students work in small groups.

SETUP



1. Avoid using a soft or felt-covered ball such as a tennis ball, since pulses from the CBR 2 tend to be absorbed by these surfaces.
2. Racquetballs work well. If you have trouble, try using a bigger ball like a basketball or smooth playground ball.
3. This activity is best performed with at least three students: one to hold the CBR 2 and press the trigger, one to release the ball, and one to run the calculator.
4. Set the sensitivity on the CBR 2 to Normal.
5. Link the CBR 2 directly to the TI-84 Plus. You can use either the I/O unit-to-unit cable or the mini-USB cable.

6. The EasyData App will launch automatically if using the mini-USB cable. If using the I/O unit-to-unit cable you will need to press the [APPS] key, scroll down to highlight the EasyData App, and press [ENTER] in order to launch the App.

7. Position the CBR 2 motion detector at least 15 centimeters above the ball (50 cm if you are using the older CBR). Hold the sensor directly over the ball and make sure that there is nothing in the ***Clear Zone***. Information about the ***Clear Zone*** can be found in the Helpful Hints section in the front of the book.

8. For best results, hold the sides of the ball and then quickly move your hands outward to release the ball.

9. Begin with a test bounce. Drop the ball (do not throw it).

DATA COLLECTION

1. Press the [Y=] key to access the **File** menu and select **1:New** by pressing [1] or since **1:New** is highlighted, you can just press [ENTER]. This resets the program and clears out old data. **See Figure 1.**

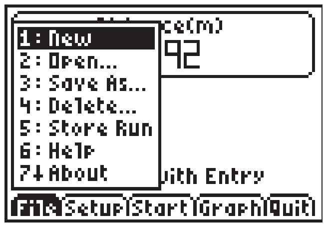


Figure 1

2. The default unit of measurement on the EasyData App is meters. You may want to do this activity in feet. To change the units of measurement, select the **Setup** menu soft key by pressing the [WINDOW] key on the top row of the calculator. From the **Setup** menu, choose **1:Dist** by pressing [1] or [ENTER] since 1 is highlighted. **See Figure 2.**

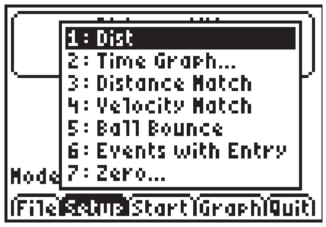


Figure 2

3. You may choose to do this activity in either meters or feet. The sample data is in meters. Press the [WINDOW] key on the top row of the calculator to access the **Units** menu. From the **Units** menu, select **1:(m)** or **2:(ft)** by pressing [1] or [2] or by scrolling until your choice is highlighted and pressing [ENTER]. Then press the [GRAPH] key on the top row of the calculator to select **OK**. **See Figure 3.**

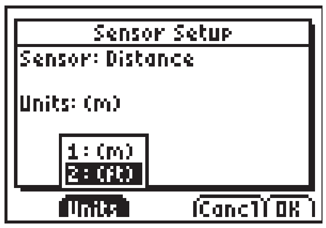


Figure 3

4. You will be returned to the main screen of the EasyData App. From the **Setup** menu, select **5:Ball Bounce** and then select **Start**. Follow the general instructions displayed. **Ball Bounce** automatically takes care of the settings. **See Figure 4.**

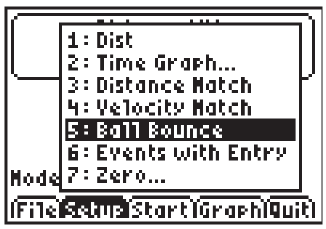


Figure 4

5. Have one person hold the CBR 2 motion detector, while another person holds the ball at least 15 centimeters beneath the sensor (50 cm if you are using an older CBR). Select **Next**. **See Figure 5.**

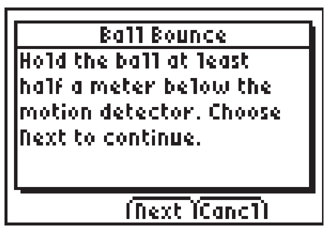


Figure 5

6. Follow the on-screen directions. At this time you may disconnect the CBR 2 from the calculator or you may leave it connected. Whether you leave it connected or not, you will need to press the **TRIGGER** on the CBR 2 motion detector to begin collecting data. **See Figure 6.**

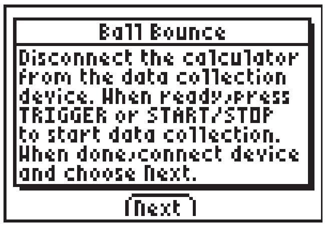


Figure 6

7. When the CBR 2 begins clicking, release the ball, and then step back. The program will collect data for 5 seconds. (If the ball bounces to the side, move to keep the CBR 2 directly above the ball, but be careful **not** to change the height of the CBR 2 motion detector.) When the clicking stops, re-connect the CBR 2 to the calculator and select **Next** on the calculator screen. The collected data is transferred to the calculator. A screen displays a notice to wait as it is transferred.

8. As soon as the data is sent, the calculator displays the distance vs. time graph from within the program. The plot should look like a bouncing ball. If it does not, repeat the sample, ensuring that the CBR 2 motion detector is aimed squarely at the ball. **See Figure 7.**

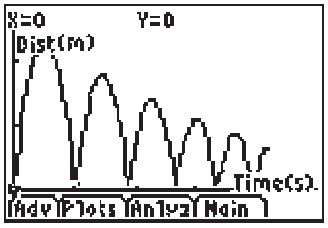


Figure 7

9. To repeat the sample, select **Main**, **Start**, and repeat the process. You will get a warning screen telling you the new data will overwrite any previous data. Select **OK. See Figure 8.**

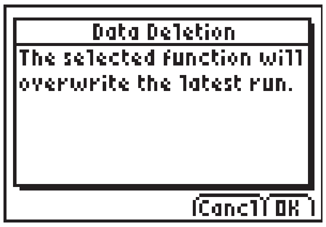


Figure 8

10. When you are satisfied with your data, study the plot. Select **Plots** to confirm a description of the graph as **1: Dist vs Time**. **See Figure 9.**

11. Guide your students in a discussion to help them realize that when the ball is at its highest point, its distance from the CBR 2 is the smallest. Observe that the **Ball Bounce** feature automatically flipped the distance data so the graph's appearance resembles the up and down movement of the ball.

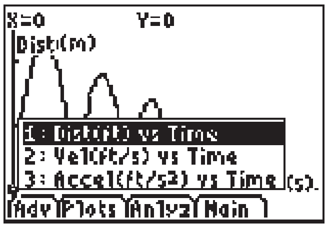


Figure 9

12. At this time, you can use the right and left arrow keys (not the TRACE key) to view the coordinates of the points. Pressing TRACE at this point will take you out to the Main screen of the EasyData App. If the students do that by accident, they can select **Graph** to return to the Graph screen. Have the students stop here and answer questions 1–7 on their worksheets.

DATA ANALYSIS

You have several options for analyzing the data. Three options will be shown here. Select the one most appropriate for your students or do all three and compare the answers. There is value in showing students different ways to accomplish any task. Consider going through option C together and then have the students repeat the process on their own in the "**Going Further**" section of their worksheet. The three methods are as follows:

A. Analyze the data from within the App. (Students will need to work in groups for the whole process as there will be only one calculator with the data.)

B. Exit the App and let the calculator identify a regression equation. (Link **L1** and **L6** so each student has his/her own set of data.)

C. Exit the App and use the vertex form of the quadratic to allow students to more thoroughly examine the relationship between quadratic equations and their parabolas. (Link **L1** and **L6** so each student has his/her own set of data.)

A. Analyze the data from wthin the App

1) Select **Anlyz** and choose **7:Select Region**. **See Figure 10.**

Figure 10

2) A warning screen is displayed. You will lose your original data. If you want to keep the original data, exit the App at this point and go to choice B or C described below. **See Figure 11.**

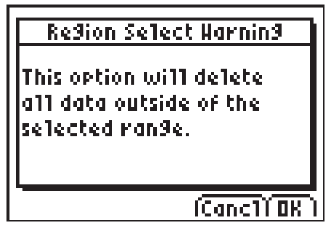


Figure 11

3) If you continue with this option, your graph is displayed and the calculator is asking you to **Set a Left Bound** for the region you wish to examine. You may want to have different groups select different regions. The example will select the second parabola. Arrow to the left end of the second parabola and select **OK**. **See Figure 12a.**

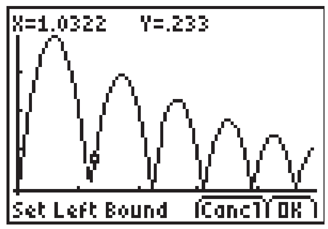


Figure 12a

CAUTION Students are familiar with pressing the ENTER key to make a selection on a graph screen. Remind them that from within this App, they need to press the GRAPH key on the top row of the calculator to access the soft key for the **OK**.

4) You will see a vertical line drawn at that point and the sentence at the bottom of the screen has changed and is asking you to **Set a Right Bound**. The cursor now appears on the far right side of the screen. Use the left arrow key to scroll to the right end of the second parabola and select **OK**. **See Figure 12b.**

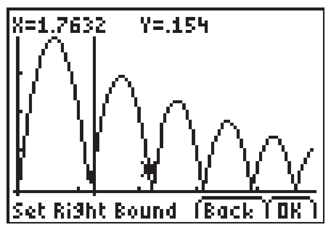


Figure 12b

5) The region you selected is drawn on the screen alone. It is clearly a parabola. **See Figure 13.**

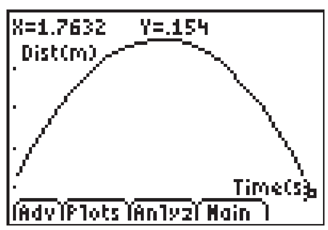


Figure 13

6) Select **Anlyz** and choose **3:Quadratic Fit** from the menu.
See Figure 14.

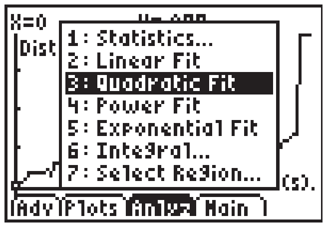


Figure 14

7) The regression equation is displayed. **See Figure 15.**

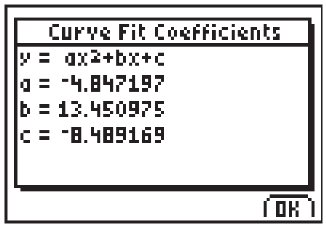


Figure 15

8) Select **OK** to see how closely the equation fits the data. The graph style has been adjusted to see the ball tracking the path. The plots have been displayed with the little crosses rather than the plain dots.
See Figure 16.

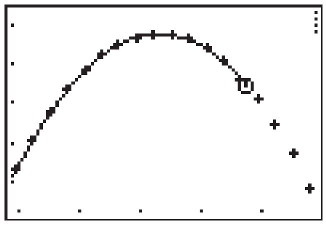


Figure 16

9) The advantage to this method is a quick and accurate answer. The disadvantage is the original data outside the selected region was lost and there was not much done in the way of concept development. **See Figure 17.**

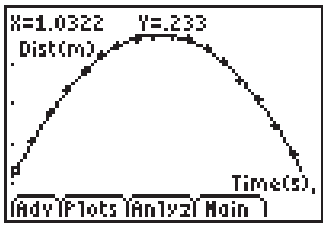


Figure 17

B. Exit the App and let the calculator identify a regression equation.

1) Once you see the data graphed, select **Main** and then **Quit**.
See Figure 18.

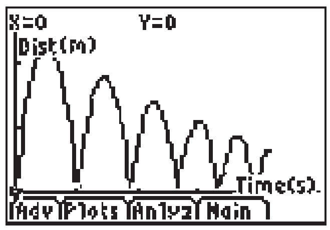


Figure 18

2) The App will display a screen telling you where the data is. For **Ball Bounce**, the time is in **L1**, the distance in **L6**, velocity in **L7**, and acceleration in **L8**. **See Figure 19.**

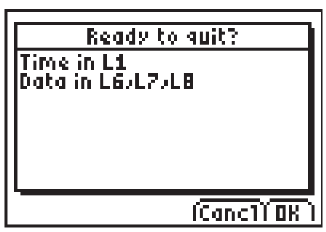


Figure 19

3) If you press [2nd] [STAT PLOT] you will see that the **Plot1** is turned on with **L1** and **L6** in the **Xlist** and **Ylist,** respectively. **See Figure 20.**

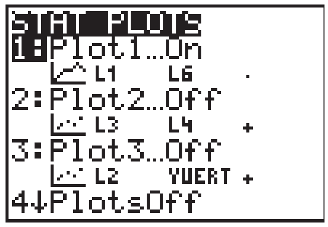


Figure 20

4) Press [GRAPH] and you will see your data displayed in the same window as when it was in the App. You can go through the same process that the App did in option A. Select a region and find a regression equation for it. **See Figure 21.**

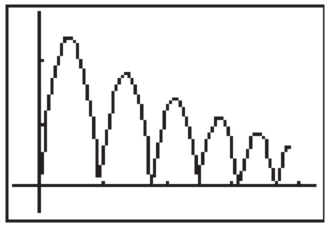


Figure 21

5) The **Select** feature on the calculator allows you to select a region from a Stat Plot you have turned on as a **Scatter** plot or **xyLine** plot. It will allow you to leave the original data in place and put the selected data in whichever lists you choose. To do this, press [2nd] [LIST] and arrow over to **OPS**, down to **8:Select** and press [ENTER]. **See Figure 22.**

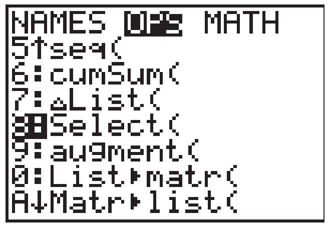


Figure 22

6) On the home screen, press [2nd] [2] [,] [2nd] [3] [)] to enter **L2** and **L3** as the arguments for the **Select** command. This is where the data points from the region you select will be stored. The nice part about doing it this way, as opposed to being inside the App, is that the original data remains intact in case you want to go back to it. This requires the new lists to have different names from the lists in which the original data is stored. Press [ENTER]. **See Figure 23.**

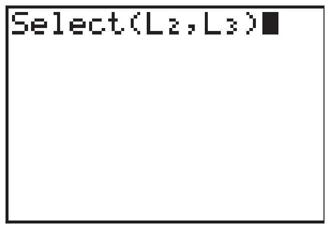


Figure 23

7) You will be taken to the graph screen and asked to select a **Left Bound**. In the example, the second parabola is being selected. Use the right arrow key to scroll over to the far left of the chosen parabola and press [ENTER]. **See Figure 24.**

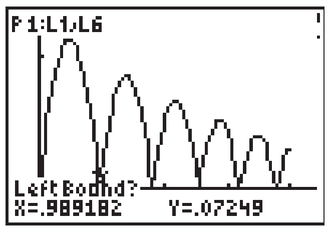


Figure 24

8) That boundary will be marked with an arrow at the top of the screen and the question at the bottom of the screen will now ask for a **Right Bound**. The cursor remains in the position where you left it. Use the right arrow key again to scroll to the far right side of the parabola and press [ENTER]. **See Figure 25.**

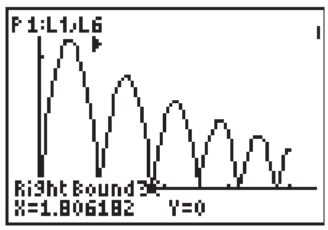


Figure 25

9) A new graph, with just the selected area, will be drawn. Press TRACE and look in the upper left corner of the screen to see that your data is in **L2** and **L3** and they have been used as the **Xlist** and the **Ylist** of **Plot1**. The selected data is always put in the same **Stat Plot** as the plot of the original data. **See Figure 26.**

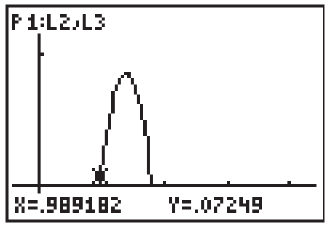


Figure 26

10) Press STAT, arrow over to **CALC**, and select **5:QuadReg**.
See Figure 27.

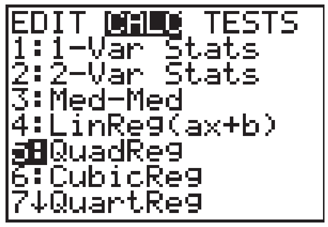


Figure 27

11) You will be taken to the home screen to enter the list names and where you want the equation pasted. Press 2nd 2 , 2nd 3 to enter **L2**, **L3**. Press the comma key again and then press VARS ▸ to access **Y-VARS** and select **1:Function**. From the list displayed, select **1:Y1**. Press ENTER to execute the command. **See Figure 28.**

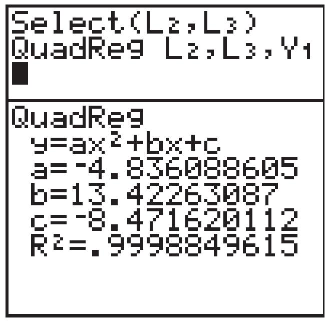


Figure 28

12) Go to Y= and move the cursor in front of **Y1** to highlight the slash icon. This controls the style of the line used to graph the equation. Press ENTER repeatedly until the graph style is the ball tracker as shown on the right here, the one with the ball and the small line to the left of the ball.
See Figure 29.

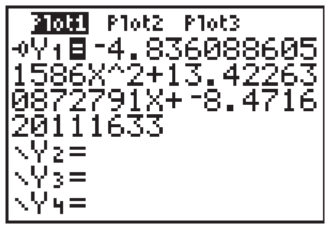


Figure 29

13) Press GRAPH to see the regression equation match the plots.
See Figure 30.

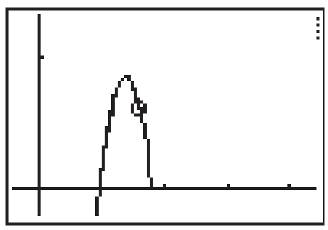


Figure 30

NOTE Think about your goals for the lesson. Both of the previous ways of obtaining the regression equation involve knowing the right keystrokes to get “the answer.” They are quick and accurate but involve very little concept development. The following method will do more for helping students learn the vertex form of a quadratic equation. It will not only help them learn where the **X**- and **Y**-values of the vertex are found in the equation, but it will also help them see how the value of A affects the shape of the parabola.

C. Exit the App and use the vertex form of the quadratic to allow students to more thoroughly examine the relationship between quadratic equations and their parabolas.

1) Begin this method the same as the method above for steps one through nine. This will have the single parabola plotted in **Plot1** with the data coordinates stored in **L2** and **L3**. Press [GRAPH]. **See Figure 31.**

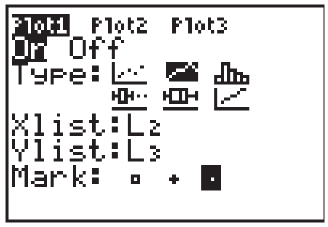


Figure 31

2) When the data is displayed in the graph window, press [TRACE] and scroll to the vertex. The **X**- and **Y**-coordinates displayed at the bottom of the screen will remain in the memory of the calculator until you trace to a new point or store new values in **X** and **Y**. **See Figure 32.**

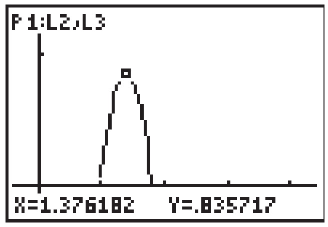


Figure 32

3) Press [2nd] [MODE] to access [QUIT] and return to the home screen. Store the coordinates from the vertex in the standard variables, **H** and **K**. Select an initial value for **A**. Hopefully your students are familiar enough with parabolas before you start this activity to know that **A** must be negative. In this case, -1 is a good place to start. The keystroke sequence is [X,T,Θ,n] [STO▸] [ALPHA] **H** [ENTER], then [ALPHA] **Y** [STO▸] [ALPHA] **K** [ENTER], and then -1 [STO▸] [ALPHA] **A** [ENTER]. **See Figure 33.**

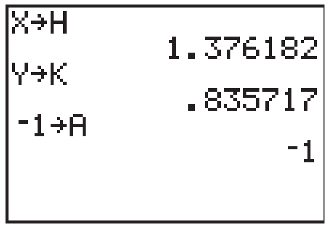


Figure 33

4) Press [Y=] and type in $\mathbf{A(X-H)^2+K}$ for **Y1**. **See Figure 34.**

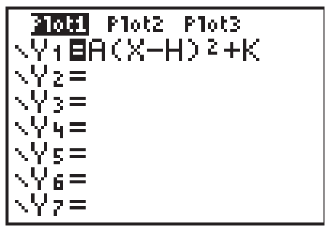


Figure 34

5) Press [GRAPH] to see how closely your equation fits the data. This step allows the students to see the vertex is in place and just the **A**-value needs adjusting. Allow them to try several values on their own as they search for a good fit. **See Figure 35.**

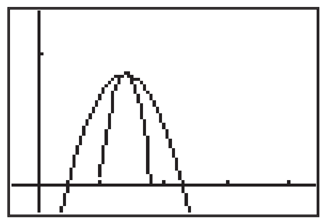


Figure 35

6) Examine the values that the students decide are a good fit for **A**. Lead them into a discussion about the acceleration of falling objects due to gravity. Remember, the ball was not thrown; it was dropped. After an object is released, it is acted upon by gravity (neglecting air resistance). So **A** depends on the acceleration due to gravity, -9.8 meters/second^2 or -32 feet/second^2. The negative sign indicates that the acceleration is downward. The value of **A** is approximately one-half the acceleration due to gravity, or -4.9 meters/second^2 or -16 feet/second^2. Depending on their knowledge of physics, decide how far you want to take this discussion. **See Figures 36a–b.**

7) Have the students store the appropriate value in **A** and press GRAPH to examine the fit.

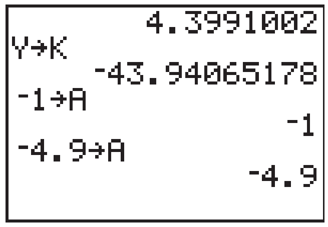


Figure 36a

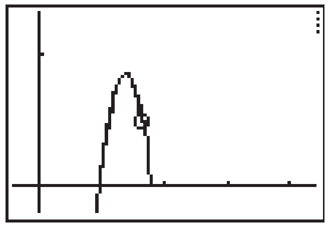


Figure 36b

NOTE You will need this data for Activity 13. To protect it from being accidentally thrown away, store **L1** and **L6** in new lists with unique names. Store **L1** in a list called **BTIME** (Ball Time) and **L6** in a list called **BDIST** (Ball Distance).

NOTE *For help with naming lists, see Appendix E.*

WORKSHEET ANSWERS

Answers will vary for problems 8, 9, 10, 12 and 20.

1. Time

2. Seconds

3. Distance/height

4. Feet or meters

5. Height of first bounce

6. The floor

7. Time was increasing. The **X**-axis is time, not distance. As time increases and you move across the **X**-axis, the ball goes up and down.

11. Approximately -4.9 if using meters, or -16 if using feet

13. + opens up,
– opens down

14. Increasing |A| will make it steeper.

15. Decreasing |A| will make it wider.

16. Different vertex

17. Same; student explanations will vary depending on their previous experiences.

18. Same or very close

19. The value of **A** is relative to gravity and therefore the same for any ball.

21. y = -4.9(x – 7) + 0.48

22. No, it would only affect the vertices of the bounces.

ACTIVITY 12

Name: ______________________

Bouncing Ball

Math Objectives:

- Graph scatter plots
- Graph and interpret a quadratic function
- Apply the vertex form of a quadratic equation
- Calculate the maximum value of a parabola

Materials:

- TI-83/TI-84 Plus Family
- Calculator-Based Ranger™ (CBR 2™)
- Vernier EasyData™ Application
- Bouncing ball

In this experiment, you will collect the height vs. time data of a bouncing ball by using the CBR 2 and the EasyData App. Your teacher will explain the procedure. When you see the EasyData App graph the ball data, complete questions 1–7.

1. What physical property is represented along the **X**-axis? ______________________
2. What are the units? ______________________
3. What physical property is represented along the **Y**-axis? ______________________
4. What are the units? ______________________
5. What does the highest point on the plot represent? ______________________
6. What does the lowest point represent? ______________________
7. Why does the plot look like the ball bounced *across* the floor. ______________________

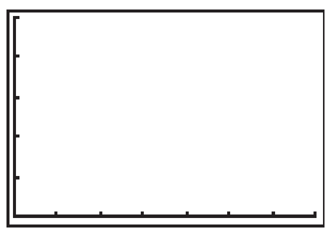
After exiting the App, use the **Select** feature on your calculator to isolate any one bounce you choose. Have the selected data put in **L2** and **L3**. For any one bounce, a plot of height vs. time has a parabolic shape. The equation that describes this motion is quadratic: $Y = A(X - H)^2 + K$ where A affects the width of the *parabola* and (H, K) is the *vertex* of the parabola.

8. This equation is called the *vertex form* of the quadratic equation. Trace along your height vs. time plot. Identify the vertex of the bounce you chose and record the **X**- and **Y**-coordinates as H and K here. H = ______________ K = ______________

   Go back to the home screen and store these values in H and K on the calculator. The keystroke sequence is [X,T,Θ,n] [STO▸] [ALPHA] **H** [ENTER] and then [ALPHA] **Y** [STO▸] [ALPHA] **K** [ENTER]. Also store –1 in the variable A by pressing **–1** [STO▸] [ALPHA] **A** [ENTER].

9. Press [Y=] and enter the expression $\mathbf{A(X - H)^2 + K}$ for **Y1** and press [GRAPH]. Sketch both your selected plot and the graph of the equation when A = –1 on the coordinate axes provided here.

10. State which bounce you worked with. ______________________



11. To find the equation of the parabola, use a guess-and-check method to find the value of A. Starting with an initial guess of A = –1 from above, adjust A by storing new values in A on the home screen. For each new value of A that you test, press [GRAPH] to view the new parabola. Experiment until you find one that provides a good fit for the data. Record the value of A that works best in the space here. A = ________

12. Using this value of A and the H and K values found in question 1, write the vertex form of the quadratic equation here. Y = ______________________________

13. What effect does the sign (positive or negative) of A have on the parabola? ______________________________

____

____

14. What effect does increasing the size of |A| have on the shape of the parabola? ______________________________

____

15. What effect does decreasing the size of |A| have on the shape of the parabola? ______________________________

____

16. How would the equation change, if at all, with a different bounce of the parabola? ______________________________

____

17. Would you expect your classmates to have the same value of A for their trials or do you think the A value would vary? Explain your answer. ______________________________

____

____

18. Find out the value of A from the other groups of students in your class. How do these values compare to your value of A? ______________________________

____

19. What conclusion can you make about the value of A of a quadratic equation for a bouncing ball? ______________________________

____

____

Going Further *Answer these questions. Show all work.*

20. Re-plot **L1** and **L6** and repeat the procedure for one of the other bounces of the original data. When you use the **Select** feature, choose **L4** and **L5** to store your data. Find the equation of this new bounce. Then, type it in **Y2** of the [Y=] window and graph it to see how well it matches the scatter plot and write it here.

____

21. Using what you discovered about the value of A in a quadratic equation for a bouncing ball, write the equation of a parabolic ball bounce with a vertex of (7, 0.48). Assume the data was measured in meters.

____

22. If a ball that was more or less bouncy was used this time, would it affect the value of A in the equation? If so, describe how.

____

____

NOTE You will need this data for Activity 13. To protect it from being accidentally thrown away, store **L1** and **L6** in new lists with unique names. Store **L1** in a list called **BTIME** (Ball Time) and **L6** in a list called **BDIST** (Ball Distance).

NOTES

