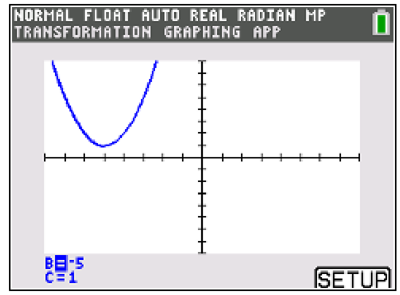
Just Move It



Math Objectives

- Students will further discuss the idea of transformations and compare a transformed function to its parent function (x^2 ***and*** x^3), both graphically and algebraically.
- Students will discuss the difference between vertical and horizontal stretches and compressions.
- Students will try to make a connection with how to understand these topics in IB Mathematics courses and on their final assessments.

Vocabulary

- Translation
- Dilation
- Compression
- Stretch
- Reflection

About the Lesson

- This lesson is aligning with the curriculum of IB Mathematics Applications and Interpretations HL and IB Mathematics Approaches and Analysis SL/HL
- This falls under the IB Mathematics Content Topic 2 Functions: **2.8** (AI HL only) and **2.11** (AA SL/HL):

  (a) Translations $y = f(x) + b$; $y = f(x - a)$

  (b) Reflections: in the x-axis $y = -f(x)$; in the y-axis $y = f(-x)$

  (c) Vertical stretch with a scale factor *p*: $y = pf(x)$

  (d) Horizontal stretch with a scale factor $\frac{1}{q}$: $y = f(qx)$

  (e) Composite Transformations

  As a result, students will:

  - Apply this information to real world situations.

Teacher Preparation and Notes.

- This activity is done with the use of the TI-84 family as an aid to the problems.

Activity Materials

- Compatible TI Technologies: TI-84 Plus*, TI-84 Plus Silver Edition*, TI-84 Plus C Silver Edition, TI-84 Plus CE

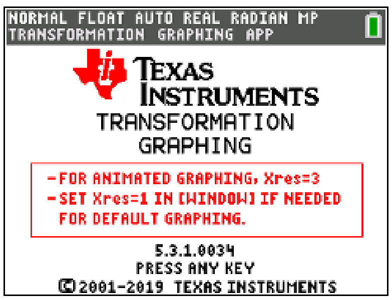
** with the latest operating system (2.55MP) featuring MathPrint™ functionality.*

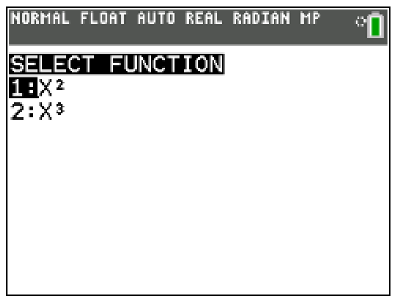
Tech Tips:

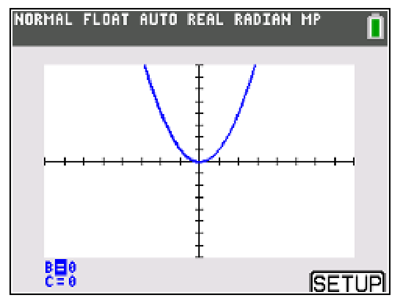
- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:
Student Activity
Just Move It_Student-84CE.pdf
Just Move It_Student-84CE.doc
MOVEIT.8xp

In this activity, the movements of the parent functions $f(x) = x^2$ and $f(x) = x^3$ will be explored. You will be using the **Transformation Graphing App** and the program **MOVEIT**, downloaded by your teacher. First, access the App by pressing **apps** and selecting **Transfrm**. Press any key to start. You will now access the **MOVEIT** program by pressing **prgm**. If you are using a TI-84 Plus CE without Python, you will select **MOVEIT**. If you are using a TI-84 Plus CE with Python, you will press **1: TI-Basic** and then select **MOVEIT**. On the Home screen, two options will appear. Option 1 will graph the parent function $f(x) = x^2$ and option 2 will graph $f(x) = x^3$. Press **2nd [quit]** to exit a graph. Press **enter** immediately to run the program again. For each problem in this activity, look at the transformation of both types of functions.







Teacher Tip: This activity can be done without the MOVEIT program. To graph the quadratic and cubic functions, enter $\mathbf{(X – B)^2 + C}$ in Y_1 and $\mathbf{(X – B)^3 + C}$ in Y_2. Press **enter** on the = sign to choose the function to graph.

Teacher Tip: The program turns the function expression off on the graph. To turn this on after having completed the activity, press **2nd , zoom [format]**, and then press **enter** when **ExprOn** is selected.

Problem 1 – $\boldsymbol{f(x) \to f(x - b)}$

Use the left and right arrow keys to change the value of **B only**. Leave C = 0. You will need to first determine the value of B in each question.

Let's see what you remember about transforming $\boldsymbol{f(x) \to f(x - b)}$**:**
In questions **1** and **2**, describe the transformation for each graph as compared to the graph of the parent function $f(x)$, use your handheld to verify your answer.

1. $f(x - 2)$ **Solution:** Horizontal translation to the right 2 units.

2. $f(x + 5)$ **Solution:** Horizontal translation to the left 5 units.

3. In general, describe the transformation of $\boldsymbol{f(x) \to f(x-b)}$ and explain your reasoning.

Possible Explanation: When the transformation of the function is affecting the input values, the resulting transformation is a horizontal translation to the left $(negative\ b)$ or to the right $(positive\ b)$.

Teacher Tip: Teachers may use different words/language to describe these transformations. Please use your judgment with the expectations of what verbiage you will require from your students when it each transformation.

Problem 2 – $\boldsymbol{f(x) \to f(x) + c}$

Use the up and down arrow keys to toggle to **C** and then the left and right arrow keys to change the value of **C only**. Leave B = 0.

Let's see what you remember about transforming $\boldsymbol{f(x) \to f(x) + c}$**:**
In questions **4** and **5**, describe the transformation for each graph as compared to the graph of the parent function $f(x)$, use your handheld to verify your answer.

4. The graph of $f(x) + 4$ **Solution:** Vertical translation up 4 units.

5. The graph of $f(x) - 3$ **Solution:** Vertical translation down 3 units.

6. In general, describe the transformation of $\boldsymbol{f(x) \to f(x) + c}$ and explain your reasoning.

Possible Explanation: When the transformation of the function is affecting the output values, the resulting transformation is a vertical translation up $(positive\ c)$ or down $(negative\ c)$.

Problem 3 – $\boldsymbol{f(x) = (x-b)^2 + c}$

7. Describe the transformations of $f(x-7) + 6$ as compared to the parent function $f(x)$.

   Solution: The quadratic has been horizontally translated to the right 7 units and vertically translated up 6 units.

8. In general, describe the transformations of $\boldsymbol{f(x) = (x - b)^2 + c}$ when:

b and c are both positive **Possible Explanation:** right b units and up c units

b and c are both negative **Possible Explanation:** left b units and down c units

b is positive and c is negative **Possible Explanation:** right b units and down c units

b is negative and c is positive **Possible Explanation:** left b units and up c units

Teacher Tip: Some discussion time will be needed to explain that a positive *b* will look like *(x – b)* and that negative *b* will look like *(x + b).*

Problem 4 – $\boldsymbol{f(x) \to af(x)}$

In order to transform your function through the multiplication of *a*, press **Y =** and enter **AX²** next to Y_1 and **AX³** next to Y_2. Press **enter** on the **=** sign to choose the function you want to graph. Press **graph** to explore the transformations.

9. Describe the transformation of $0.5f(x)$ as compared to the parent function $f(x)$.

Solution: This is a vertical compression by a factor of 0.5.

With a classmate, create a table of values comparing the y-values for given x-values for the functions $f(x) = x^2$ and $f(x) = 0.5x^2$. For example, when x = 2, the corresponding values for the functions are 4 and 2 respectively. In other words, the y-values are “pushed lower” as a result of multiplying by 0.5. This is known as a _____**vertical compression**________________.

10. Describe the transformation of $2f(x)$ as compared to the parent function $f(x)$.

Solution: This is a vertical stretch by a factor of 2.

11. In general, describe the transformation when $0 < |a| < 1$ for the graph of $af(x)$ as compared to the parent function $f(x)$.

Solution: A vertical compression by a factor of $|a|$.

12. In general, describe the transformation when $|a| > 1$ for the graph of $af(x)$ as compared to the parent function $f(x)$.

    Solution: A vertical stretch by a factor of $|a|$.

13. Change the coefficient of the quadratic and cubic functions to -0.5 and then to -2. Describe the graph of $af(x)$ when a is negative as compared to when a is positive.

    Solution: The function is now reflected over the x-axis.

Problem 5 – $f(x) \to f(ax)$

In order to transform your function through the multiplication of a, press **Y =** and enter **(AX)²** next to Y_1 and **(AX)³** next to Y_2. Press **enter** on the **=** sign to choose the function you want to graph. Press **graph** to explore the transformations.

14. Describe the transformation of $f(2x)$ as compared to the parent function $f(x)$.

    Solution: Horizontal compression by a factor of $\frac{1}{2}$.

With a classmate, create a table of values comparing the y-values for given x-values for the functions $f(x) = x^2$ and $f(x) = (2x)^2$. For example, when x = 2, the corresponding values for the functions are 4 and 16 respectively. In other words, instead of it taking x = 4 to get y = 16, it took x = 2 to get y = 16, therefore x-values are "pushed lower" as a result of multiplying the x-value by 2 or the x-value was halved. This is known as a ___**horizontal compression**____________.

15. Describe the transformation of $f(0.5x)$ as compared to the parent function $f(x)$.

    Solution: Horizontal stretch by a factor of 2.

16. In general, describe the transformation when $0 < |a| < 1$ for the graph of $f(ax)$ as compared to the parent function $f(x)$.

    Solution: Horizontal compression by a factor of $\frac{1}{|a|}$.

17. In general, describe the transformation when $|a| > 1$ for the graph of $f(ax)$ as compared to the parent function $f(x)$.

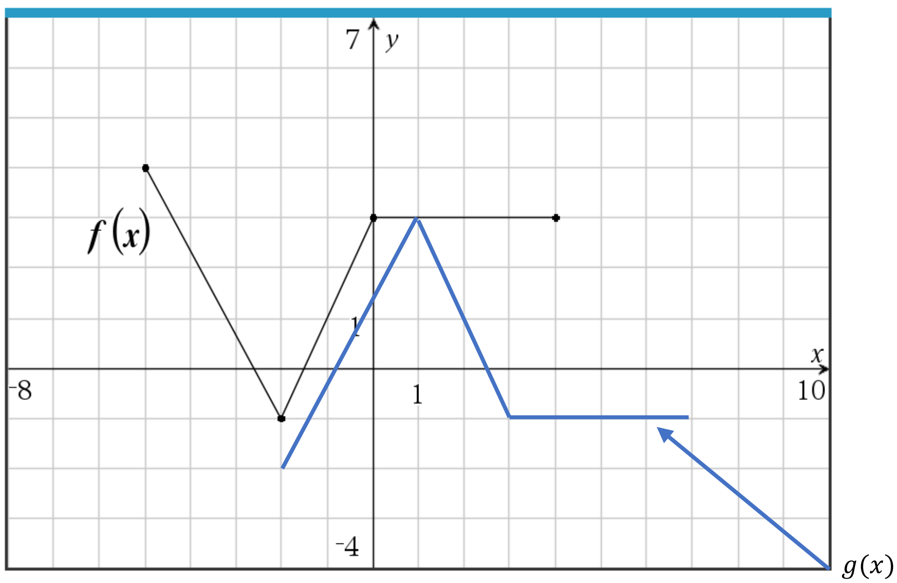
    Solution: Horizontal stretch by a factor of $\frac{1}{|a|}$.

18. Change the sign of the value being multiplied by x for the quadratic and cubic functions to -0.5 and then to -2. Describe the graph of $af(x)$ when a is negative as compared to when a is positive.

Solution: The function is now reflected over the y-axis.

Further IB Applications

The following diagram shows the graph of the function $y = f(x)$, for $-5 \le x \le 4$. The points (-5, 4) and (0, 3) both lie on the graph of f. There is a minimum at point (-2, -1).



Let $g(x) = -f(x-3) + 2$.

(a) Write down the domain of f. **Solution:** $-5 \le x \le 4 \; or \; [-5, 4]$

(b) Write down the range of g. **Solution:** $-2 \le y \le 3 \; or \; [-2, 3]$

(c) On the graph above, sketch the graph of g. **Solution:** See graph above.

Let $h(x) = f(-3x)$.

(d) Describe the transformations of $h(x)$ as compared to $f(x)$.

Solution: The function has been reflected over the y-axis and horizontally compressed by a factor of $\frac{1}{3}$.

Teacher Tip: Please know that in this activity there is a lot of time dedicated to students talking with one another and sharing their thoughts with the class. The goal here is to not only review transformations of functions, but also to generate discussion.

***Note: This activity has been developed independently by Texas Instruments and aligned with the IB Mathematics curriculum, but is not endorsed by IB™. IB is a registered trademark owned by the International Baccalaureate Organization.*