Population Mean: σ unknown

Student Activity

Name ____________________

Class ____________________

Problem 1 – Assumptions

Goal: Estimate a population mean.

- When σ is known, the normal distribution and z-scores are used.
- When σ is not known, two assumptions are made:

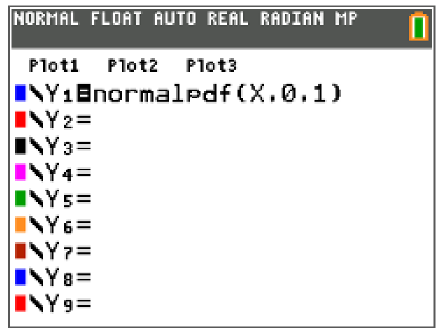
  1. It is a simple random sample.
  2. The sample is from a normally distributed population, or *n* (the sample size) > 30.

  If these assumptions are true, use a *t* distribution. For a sample size *n* of a *t* distribution, the degrees of freedom is *n* – 1.

Graph normal distribution **Y1=normpdf(X,0,1)**.

The **normpdf(** command is found in the **DISTR** menu.

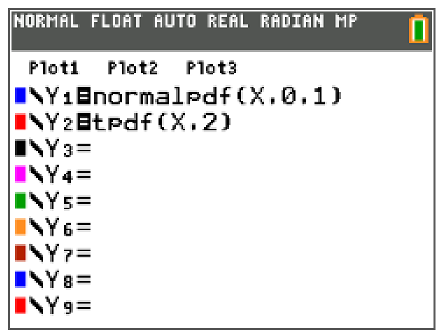
Adjust the window appropriately to view the graph.



Graph the *t* distribution for *n* = 3: **Y2=tpdf(X,2)**.

The **tpdf(** command is in the **DISTR** menu.

After viewing the graph, increase the value of *n* for the *t* distribution. View the graph after each increase.



1. What happens as $n \to 30$?

2. How does the size of the sample play a role in the accuracy of the estimation?

The following problems are properties of a simple random sample. Decide which distribution can be used to estimate the population mean and use the calculator commands shown above. Be prepared to justify your answer.

3. Determine whether to use a normal distribution, *t* distribution, or neither.

a. $n = 50$, $\bar{x} = 10$, $s = 4$, population is skewed.

b. $n = 15$, $\bar{x} = 10$, $s = 4$, population is normally distributed.

c. $n = 50$, $\bar{x} = 10$, $\sigma = 4$, population is very skewed.

d. $n = 15$, $\bar{x} = 10$, $s = 4$, population is skewed.

Problem 2 – Estimating the Interval

The true mean for the population will almost always be contained in an interval $\bar{x} \pm E$ (an error). The error is dependent upon the confidence level chosen. The larger the probability, the larger the interval.

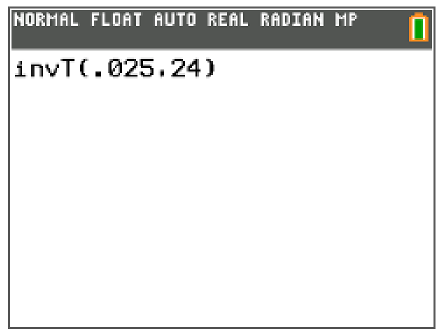
Margin of Error	Confidence Interval
$E = t_{\alpha/2}\dfrac{s}{\sqrt{n}}$	$\bar{x} - E < \mu < \bar{x} + E$

1 – α is the probability that μ (the population mean) is in interval. So, if we desire a 95% confidence interval, then α = 0.05.

4. Find a 95% confidence interval for a sample where n = 25, $\bar{x}$ = 15, and s = 0.5 and the data is normal distributed.

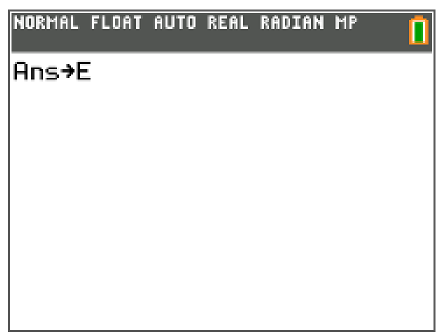
Step 1: Find $t_{\alpha/2}$. Choose the **invT(** command from the **DISTR** menu and enter the area and degrees of freedom. (Area = 0.025, because α = 0.05, and df = 24).

The t-value needed to calculate the error is the absolute value of this number. Find it by choosing **abs(** from the **MATH > NUM** menu and pressing math.

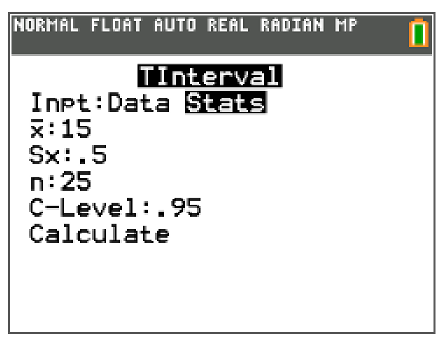


Step 2: Calculate the value of E and store it as **E** using the sto→ key.

Step 3: Find the interval: $\bar{x} - E < \mu < \bar{x} + E$.



You can use the built-in command, **TInterval** to find the intervals faster. Choose **TInterval** from the **STAT > Tests** menu and enter the information as shown. To calculate the interval, highlight **Calculate** and press enter.



Name ___________________
Class ___________________

Extension – Using Data

The data in the **JANTM** list (provided in the chart to the right as well) gives the normal average January minimum temperature in degrees Fahrenheit of 56 cities.

5. Find an interval that estimates the true population mean (average January temperatures for all US cities) with:

a. 90% confidence

b. 95% confidence

c. 99% confidence

Normal Average January Low Temperatures for 56 Cities			
44	35	31	47
38	42	15	22
26	30	45	65
58	37	22	19
21	11	22	27
45	12	25	23
21	2	24	8
13	11	27	24
14	27	34	31
0	26	21	28
33	24	24	38
31	24	49	44
18	7	32	33
19	9	13	14

