**Using the Document**

FundamentalTheorem.tns:

On page 1.2, the function  is defined in a Math Box. The default definition for  is . This expression can be changed by the user to allow for more in-depth discussions and conceptual questions concerning the Fundamental Theorem of Calculus.

The graph of  is displayed on Page 1.3. The values  and  can be changed by grabbing the corresponding point and dragging along the horizontal axis. The value  is displayed in the bottom pane.

Page 1.4 shows a graph of  and a graph of , aligned horizontally. The values  and  can be changed on the graph of  by grabbing the corresponding point and dragging along the horizontal axis. Page 1.5 shows the graph of  in the top pane, and the graph of its derivative in the bottom pane.

Problem 2 presents a special application in which the accumulation function is used to define the natural logarithm function. Pages 2.3 and 2.4 present graphical evidence to confirm the definition of the natural logarithm function and its derivative.

**Suggested Applications and Extensions**

**Page 1.3**

Use the default function  and the default value of  to answer Questions 1-5.

Remember that  is a function of  (for a fixed value of ). The values of  and  can be manipulated, the value of  is displayed in the bottom pane, and the shaded region in the top pane represents the accumulated net area bounded by the graph of  and the horizontal axis from  to .

1. Use geometry to estimate the value for . Then move the point representing  to  on the horizontal axis to find the exact value. Is your estimate for  an overestimate or underestimate? Why?
2. Use geometry to estimate the value for . Then move the point representing  to  on the horizontal axis to find the exact value. Explain why .
3. On what intervals is  increasing? Decreasing? What are the values of  on each of these intervals?
4. For what value(s) of  does  have a relative maximum? Relative minimum? Find  for each value of  at which  has a relative extrema. What does this suggest about the relationship between  and ?
5. Find the absolute maximum value and the absolute minimum value of  on the interval .
6. Let . Find  and explain why  even though there is more shaded area below the horizontal axis than above.
7. Let . Find the absolute maximum value and absolute minimum value of  on the interval . How do these answers compare with those in Question 5?
8. Let  and find . Let  and find . How do these two values compare? Is this result always true if the values of  and  are switched? Why or why not?

**Page 1.5**

Use the default function  and the default value of  to answer Questions 1-4.

The bottom pane shows a graph of the function .

1. Use the graph of  to find the intervals on which  is increasing. Decreasing. What are the values of  on each of these intervals? How do our answers compare with those in Question 3 above?
2. Estimate the slope of the tangent line to the graph of  and the value of  at . How do these values compare?
3. Find the intervals on which the graph of  is concave down. Concave up. Estimate the  of the point inflection on the graph of . What do you notice about the graph of  at this value? Explain the behavior of  around this value.
4. Find an equation of the tangent line to the graph of  at the point with  2.
5. Grab the point representing  in the top pane and move it slowly to the right, until . Describe how the graph of  changes as  increases from  to 5. Try to use the graph of  to explain how  changes.

**Page 1.7**

The graph of  is shown in the top pane, and the graph of  is shown in the bottom pane.

1. Grab and move the point in the top pane representing . Verify that the value of  is the slope of the tangent line to the graph of  at .
2. Compare the graph of  in the bottom pane with the graph of  on page 1.5 in the top pane. What does this suggest about the relationship between  and ?
3. Grab and move the point in the bottom pane representing . Explain why the graph of  changes but the graph of  does not change.
4. Can you move the point representing  (in the bottom pane) such that the slope of the tangent line to the graph of  at  is ? Why or why not?

**Pages 2.3 and 2.4**

The purpose of this example is to use the Fundamental Theorem of Calculus to define the natural logarithm function.

**Page 2.3**

1. Use the graph of  and the definition of  to explain why the graph of  is always increasing.
2. Use the graph of  to give a geometric interpretation of the value of  and .
3. Use the definition of the accumulation function to explain why .
4. Use the graph of  to describe the concavity of .
5. Find the values  and . Explain how these two values are related in terms of accumulated area.

**Page 2.4**

1. Grab and move the point in the top pane representing . Verify that the value of  is the slope of the tangent line to the graph of  at .
2. Use these graphs to confirm and state the relationship between the functions  and .
3. Use these graphs and the definition of the accumulation function to explain why  for .
4. As  increases, describe the behavior of the slopes of the tangent lines to the graph of . Use the graph of  to confirm this observation.
5. Find the slope of the tangent line to the graph of  at  and at . Explain how these values are related and why.