Infinite Geometric Series

Name ______________________

Class ______________________

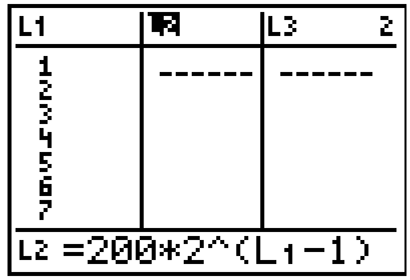
Problem 1 – Investigating Infinite Geometric Series

Explore what happens when the common ratio changes for an infinite geometric series.

For each of the *r*-values in the table, you need to

a. create lists L2 and L3
b. determine if the series converges
c. if it does converge, create and view the scatter plot

Press STAT ENTER to open the list editor.



Create list **L1** with values 1 to 20 by entering **seq(X, X, 1, 20, 1)** in the top of the L1 field.

Create list **L2** with values of $ar^{n-1} = 200r^{n-1}$ by entering **200∗r ^(L1–1)** with different values of **r** based on the table below.

Create list **L3** with the partial sums by entering **cumSum(L2)**.

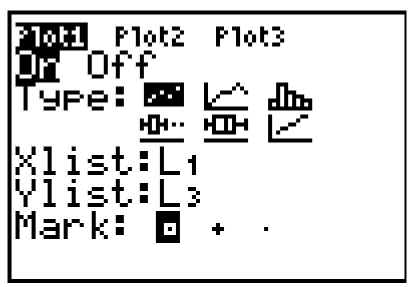
Note: Press 2nd [LIST], arrow right to the OPS menu for the **seq(** and **cumSum(** commands.

1. Look at the partial sums (in L3) and determine if the sums converge to a number or diverge to infinity (or negative infinity) as *n* gets large.

Record your results in the table. If it converges, then what does it appear to converge to?

r	–2	–0.5	–0.25	0.25	0.5	2
Converges or Diverges						

Press 2nd [STAT PLOT], and select **Plot1**. Create a scatter plot, choosing **L1** for *x* and **L3** for *y*. Press ZOOM and select **ZoomData**.



2. For what range of values of *r* does the infinite geometric series converge?

3. What do you notice about the scatter plot when the series converges?

Problem 2 – Deriving a Formula for the Sum of a Convergent Infinite Geometric Series

Recall that a finite geometric series is of the form

$$S_n = a_1 + r \cdot a_1 + r^2 \cdot a_1 + r^3 \cdot a_1 + r^4 \cdot a_1 + ... + r^n \cdot a_1 = \sum_{i=1}^{n} a_1 r^{n-1} = \frac{a_1(1-r^n)}{1-r}$$

4. Let r = 0.7. Use the Home screen to complete the following table.

n	10	100	1000	10000
$r^n = 0.7^n$				

If | r | < 1, then what is the value of r^n as n gets very large?

5. How does the formula $S_n = \frac{a_1(1-r^n)}{1-r}$ change, if n goes to infinity (gets very large)?

Therefore, if | r | < 1, then the infinite geometric series of the form

$S = a_1 + r \cdot a_1 + r^2 \cdot a_1 + r^3 \cdot a_1 + r^4 \cdot a_1 + ...$ converges and has the sum $S = \frac{a_1}{1-r}$.

Problem 3 – Apply what was learned

Use the formulas for the sums of finite and infinite geometric series to complete this problem.

6. A patient is prescribed a 240 mg dose of a long-term, pain-reducing drug that should be taken every 4 hours. It is known that after each hour, 15% of the original dosage leaves the body. Under these conditions, the amount of drugs remaining in the body (at 4-hour intervals) forms a geometric series.

a. What is the common ratio of the geometric series?

b. How many milligrams of the drug are present in the body after **4** hours (2^{nd} dosage)?

c. Complete the table for the amount of the drug in the body for several 4-hour intervals.

Hours	0 (1st dosage)	4 (2^{nd} dosage)	8 (3^{rd} dosage)	12	16
Amount in the Body					

d. How many milligrams of the drug are in the body after **24** hours?

e. How many milligrams of the drug are in the body after **72** hours?

f. How many milligrams of the drug are in the body after ***t*** hours?

g. The minimum lethal dosage of the pain-reducing drug is 600 mg. Will the patient ever have this much of the drug in his or her system if he or she continuously (infinitely) takes the drug every four hours?

h. If the patient decides to take the drug every 2 hours, against the doctor's orders, then will the patient reach the minimum lethal dosage?