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Exploring the Parabola

TIMATH.COM: PRECALCULUS

Activity Overview

In this activity, students explore the key features of the parabola, both geometrically and algebraically. A variety of interactive representations support student learning as they build their understanding of this important curve and its real-world applications.

Topic: Conics

- Write the equation of a parabola with vertex at (*h*, *k*) and axis of symmetry x = h or y = k and graph it.
- Derive the equation of any parabola (or conic) using the focusdirectrix definition.

Teacher Preparation and Notes

- Students should already be familiar with the features that make the parabola both unique and important. This activity supports students in actively linking some of the geometric and algebraic properties of a parabola.
- This activity is designed to be teacher-led with periods of small group work.
- It is useful to have students work in pairs, both to encourage verbalization and discussion, and to offer scaffolding when needed. Individuals may work together with their own handhelds, or they may share with partners. Each must keep a record of the process, documenting his/her own observations and any questions that arise.
- To download the student worksheet, go to <u>education.ti.com/exchange</u> and enter "10036" in the keyword search box.

Suggested Related Activities

To download any activity listed, go to <u>education.ti.com/exchange</u> and enter the number in the keyword search box.

- Properties of Parabolas (TI-84 Plus family) 8854
- Parabola Construction (TI-Nspire technology) 12553
- What's My Locus? (TI-Nspire technology) 8255



This activity utilizes MathPrint[™] functionality and includes screen captures taken from the TI-84 Plus C Silver Edition. It is also appropriate for use with the TI-83 Plus, TI-84 Plus, and TI-84 Plus Silver Edition but slight variances may be found within the directions.

Compatible Devices:

- TI-84 Plus Family
- TI-84 Plus C Silver Edition

Associated Materials:

- ExploringTheParabola_Student .pdf
- ExploringTheParabola_Student .doc

Click <u>HERE</u> for Graphing Calculator Tutorials.



Problem 1 – Perpendicular Bisector

Taking a sheet of paper, students should mark a point somewhere near the center of the page. They can then produce a series of folds from different points along one of the long edges that pass through this point. The locus formed by the paper folding gives an approximation of a parabola.

It is important that students see the connection between the crease in the paper and the perpendicular bisector.

If students have trouble seeing this, have them open Cabri Jr. and start a new document. Use the **Hide/Show** tool to display the axes. Students should place one point anywhere on the *x*axis and one point in the 1st quadrant. Then, they should select the **Perp. Bis.** tool and select the two points. They can drag either point. To display the parabola, have students use the **Locus** tool and select the line and then the point on the *x*-axis.





Problem 2 – Shape of the Curve

This section deals with the shape of the curve and is leading up to the general equation for the parabola, $4p(y - k) = (x - h)^2$. The equations of the parabolas are given to the students and they are asked to fill in tables and answer some questions. The second column is the distance from the point to the focus and the third column is the distance from the point to the directrix.

Student Worksheet Solutions

Point	Distance from (2, 2)	Distance from <i>x</i> -axis
(2, 1)	1	1
(1, 1.25)	1.25	1.25
(5, 3.25)	3.25	3.25
(0, 2)	2	2

$$y = \frac{1}{4}(x-2)^2 + 1$$

<i>y</i> =	(x –	$(3)^2$	+	0.2	5
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Point	Distance from (3, 0.5)	Distance from <i>x</i> -axis
(3, 0.25)	0.25	0.25
(0, 9.25)	9.25	9.25
(2, 1.25)	1.25	1.25
(5, 4.25)	4.25	4.25



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Point	Distance from (1, -1)	Distance from <i>x</i> -axis
(1, –0.5)	0.5	0.5
(0, -1)	1	1
(-1, -2.5)	2.5	2.5
(4, –5)	5	5

$$y = -\frac{1}{2}(x-1)^2 - 0.5$$

The distances from the point to the directrix and the point to the focus are the same. This happens because the point is on the perpendicular bisector.

The smaller the coefficient, the broader the parabola. The larger the coefficient, the narrower the parabola.

The closer the focus and directrix, the narrower the parabola. The farther away, the broader the parabola.

The coefficient of the square term is $\frac{1}{4p}$ where *p* is the distance from the vertex to the focus.

Problem 3 – Fitting a Parabola

Students will use the information gathered from the tables to answer the questions in this section.

1. What is the equation of the parabola with focus at (0, 4) and directrix on the x-axis?

$$y=\frac{1}{8}x^2+2$$

2. What is the equation of the parabola with the focus at (2, 3) and directrix at y = 0?

$$y = \frac{1}{6}(x-2)^2 + \frac{3}{2}$$

3. What is the focus for the parabola with equation $y = 0.125(x - 3)^2 + 2$?

(3, 4)

4. What is the significance of the coefficient of *x* in each of these equations?

The coefficient is given as $\frac{1}{4p}$ where *p* is the distance from the focus to the vertex.

5. Can you find the general form for the equation of a parabola with its focus located at (*a*, *b*) and directrix on the *x*-axis?

$$y=\frac{1}{2b}(x-a)^2+\frac{b}{2}$$