Projectile Trajectories
Student Activity

Name ______________

Class ______________

Open the TI-Nspire document *Projectile_Trajectories.tns.*

What determines the trajectory of a projectile? What happens to its motion during its flight? How can different shots, following different paths from the same starting point, still end up in the basket? We can investigate the basics of projectile motion using vectors and parametric equations to define and describe the motion.

A projectile travels both horizontally and vertically at the same time. These two simultaneous motions can be combined using vectors,and can be described separately in a set of parametric equations.

Let's begin by looking at a simple projectile launched upwards from "ground level" across a horizontal surface. Start with the initial velocity and assume no air resistance.

Press ctrl ▶ and ctrl ◀ to navigate through the lesson.

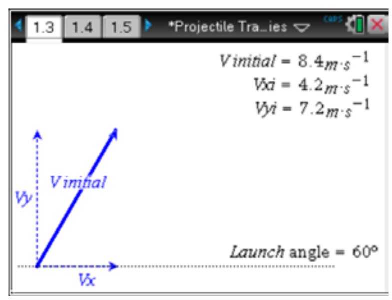
Move to 1.2.

Problem 1: Projectile Motion Components

The trajectory, or path, of a projectile depends on the initial velocity vector. This initial velocity can have different magnitudes and directions. Varying the initial velocity vector causes its *x* (or horizontal) and *y* (or vertical) components to change.

Move to page 1.3.

1. Drag the tip of the initial velocity vector and note how the launch angle and the components ***Vx*** and ***Vy*** change.



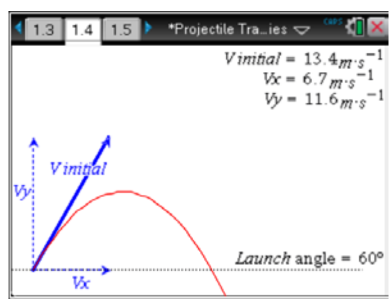
Move to page 1.4.

2. Here is a projectile trajectory determined by the initial velocity. Drag the tip of the initial velocity and note how the trajectory changes.

   Press **Menu > Trace > Graph Trace**.

   Move the trace point along the graph to see the values of *x* (horizontal distance), the *y* (vertical distance), and *t* (time of flight) for each point on the trajectory.

   Click [click] on the trace point to lock it, giving the *x* and *y* coordinates of that point. Press esc to close the Trace tool. The trace point can then be dragged.



Move to pages 1.5–1.9. Answer the following questions here or in the .tns file.

Q1. What two things tend to make the projectile travel <u>higher</u>?

Q2. What two things can you do to make the projectile travel <u>farther</u>?

Q3. Set a suitable trajectory (between 20° and 70°) on page 1.4. What is the maximum height of that trajectory? Lock in a trace point.

Q4. When did the projectile reach this maximum height, and what was its horizontal displacement at this time?

Q5. At what time and location did the projectile land on the ground?

Move to page 2.1.

Problem 2: Horizontal and Vertical Motions

As a projectile flies through the air, its motion changes. Vertical and horizontal motions are separate and independent. Each has its own equation to describe it. For simple projectile motion (ignoring air resistance, Earth movement, curvature of the Earth, etc.), horizontal motion is uniform and based on ***Vx*** (horizontal component of the initial velocity). The vertical motion is controlled by gravity and based on ***Vy*** (the vertical component of the initial velocity).

Move to pages 2.2 and 2.3. Answer the following questions here or in the .tns file.

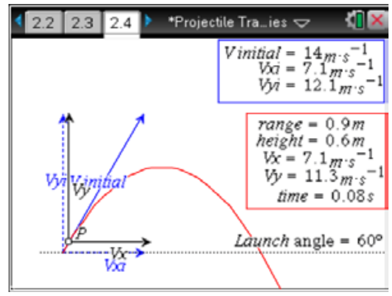
Q6. What is the equation to determine distance for uniform <u>horizontal</u> motion?

Q7. What is the equation for <u>vertical</u> height in free-fall motion?

Parametric equations relate two separate relations to a third variable—usually time. This works perfectly for projectiles; we can determine horizontal and vertical displacements from different velocities—both at the same times. The parametric equations for vertical and horizontal motions are stepped through chosen time intervals to create the trajectories in this document.

Move to page 2.4.

3. Here is a trajectory that you can vary by changing the initial velocity. The projectile is attached to the trajectory path and the velocity of the projectile is shown as you drag P along the trajectory.



Move to pages 2.5–2.8. Answer the following questions here or in the .tns file.

Q8. What happens to the magnitude of the <u>horizontal</u> component of velocity (**Vx**) as the projectile moves along the trajectory?

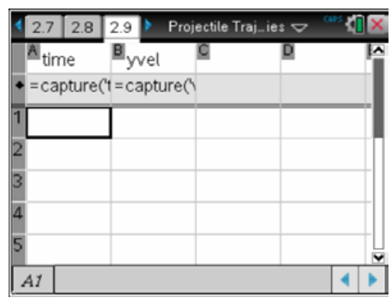
Q9. What happens to the magnitude of the <u>vertical</u> velocity (**Vy**) as the projectile moves along the trajectory?

Q10. When does the vertical velocity become zero? Why? What is this point of the trajectory called?

Q11. What happens to the vertical velocity (**Vy**) after the projectile reaches maximum height? Explain.

Move to page 2.9.

This is a spreadsheet with columns for the **time** and vertical velocity (**yvel**) component as the projectile moves along the trajectory. The spreadsheet is set up to capture data from your trajectory.



4. Go back to page 2.4, set a suitable trajectory and move the projectile near the beginning of the trajectory.
   Press ctrl . . This captures data from this point for the spreadsheet.
   Move the projectile along the trajectory and press ctrl . to gather another set of data. Do this for <u>at least ten points</u> along the range of the trajectory.
   Look at the spreadsheet (page 2.9), then move on to page 2.10.

Move to page 2.10.

You have captured data of time and *y*-velocity along the trajectory. By plotting this data, you can find how the *y*-velocity changes with time.

5. Press tab until the x-axis options are shown; select the variable **time**. Press tab again until the y-axis options are shown; select the variable **yvel**. This plots the captured data from the spreadsheet. Press **Menu > Analyze > Regression > Show Linear (mx+b)**. Note the equation that appears.

Move to pages 2.11–2.21. Answer the following questions here or in the .tns file.

Q12. What does the slope of any velocity vs. time graph represent?

Q13. What is the slope of this equation?

Q14. What units should the slope have?

Q15. Explain what this slope means.

Q16. What is the vertical axis intercept (*y*-intercept)? What does this mean?

Q17. How does the *y*-intercept relate to the original trajectory (on page 2.4)?

Q18. What does the point where the line crosses the *x*-axis represent? Where is this seen on the trajectory?

Q19. What happens to the vertical velocity at the point where the graph crosses the *x*-axis? Explain what is going on here.

Q20. Why is the vertical acceleration negative throughout the entire trajectory?

Q21. Does horizontal motion affect the vertical motion?

A. Yes B. No

Q22. Does vertical motion affect the horizontal motion?

A. Yes B. No

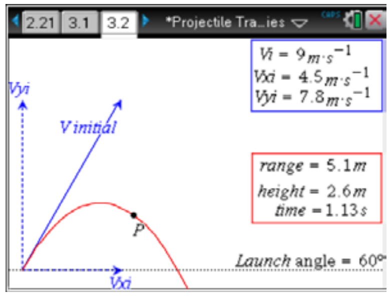
Problem 3: Maximize Range & Optimum Angle

Move to page 3.1. Answer the following question here or in the .tns file.

Q23. How can you maximize the range of a projectile? Shooting or throwing it faster is obvious; what else can you do?

Move to page 3.2.

6. Here is a trajectory for a projectile launched at a fixed, maximum speed. Explore how to increase the range of the projectile. You can drag the initial velocity vector, and you can drag the trace point on the trajectory.



Move to page 3.3.

Q24. In this example, what launch angle gives the maximum range?

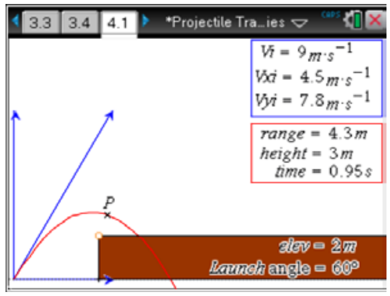
Move to page 3.4.

Q25. What is the maximum range?

Problem 4: Elevated Landing

Move to page 4.1.

7. You now launch the projectile towards an elevated landing area. Find the optimum launch angle by dragging the tip of the initial velocity vector and the trace point. Explore the affect by changing the elevated height by dragging the corner of the elevated surface and again finding the optimum launch angle.



Move to pages 4.2–4.4. Answer the following questions here or in the .tns file.

Q26. What happens to the range when the landing area is elevated above the launch location?

Q27. Does elevation of the landing area affect the optimum angle for maximum range?

A. Yes B. No

Q28. As the elevated landing area becomes higher, what happens to the angle needed for maximum range?

Extension

As an extension to this activity, consider another trajectory where the launch point is elevated above the landing area. What does this do to the range and to the optimum angle for maximum range?