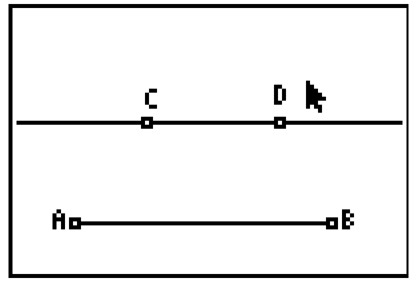
Properties of Trapezoids and Isosceles Trapezoids

Name ____________________

Class ____________________

To begin, open up a blank Cabri Jr. document. Let's start with a line segment, *AB*, and a point *C* above the segment.

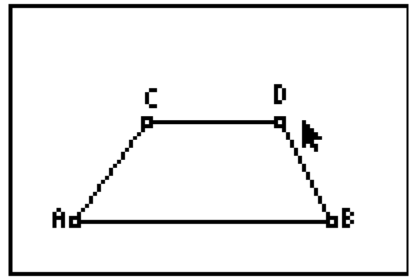
Create a line through *C* that is parallel to $\overline{AB}$. Construct a point on the new line and label it *D*.



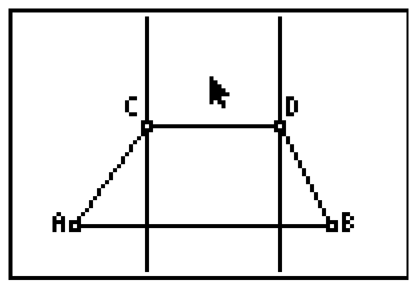
Hide the parallel line and complete the trapezoid. Measure the interior angles.

- Are the base angles at *C* and *D* congruent? Will they ever be? What about the base angles at *A* and *B*?

- Measure the lengths of the diagonals $\overline{AD}$ and $\overline{BC}$. Will they ever be congruent?

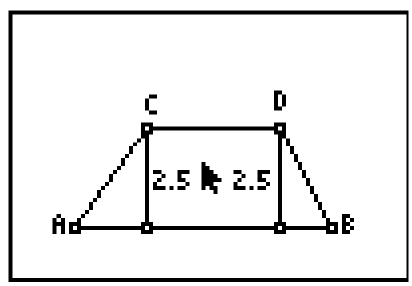


To verify that the lines are parallel, construct lines through *C* and *D* that are perpendicular to $\overline{AB}$.



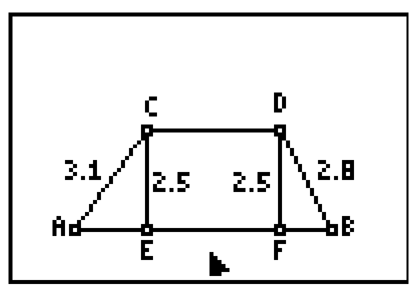
Construct the points of intersection of the new lines with $\overline{AB}$. Construct lines segments to connect *C* and *D* to the line through $\overline{AB}$ and measure the lengths of these segments.

- Will these segments always be equal? Should they be? Why?



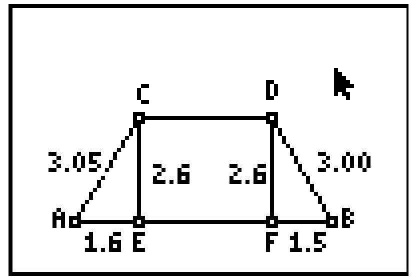
Label the new points on $\overline{AB}$ as *E* and *F* and measure the lenghts of $\overline{AC}$ and $\overline{BD}$.

- Will $\overline{AC}$ ever be congruent to $\overline{BD}$?

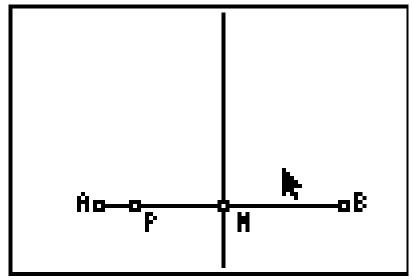


Try to drag point *C* or point *D* to make $AC = BD$. Due to the screen resolution, this can be very difficult. Construct $\overline{AE}$ and $\overline{BF}$.

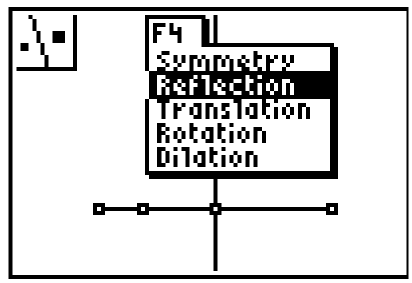
- For an isosceles trapezoid, these segments should also be congruent. Can you explain why?



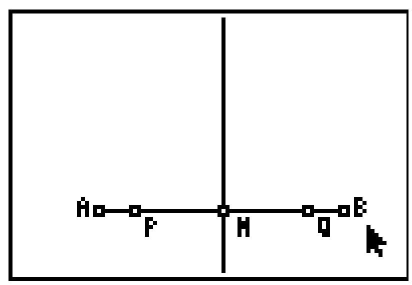
In order to construct an isosceles trapezoid, start with a line segment, *AB*. Construct the midpoint, *M*, and another point, *P*, on $\overline{AB}$. Construct a line through the *M* that is perpendicular to $\overline{AB}$.



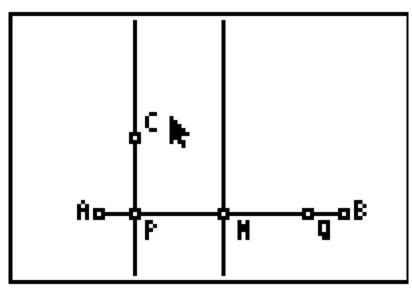
Press TRACE and select the Reflection option. Click on the perpendicular line through *M* and then point *P*.



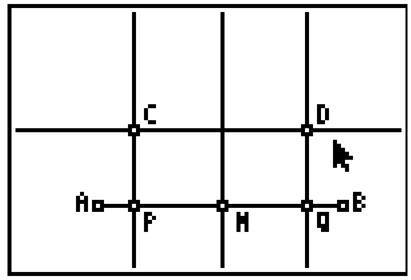
A new point will appear on the line segment on the right side. Label this point *Q*.



Construct perpendicular lines through *P* and *Q*. Construct point *C* on the perpendicular through *P*.

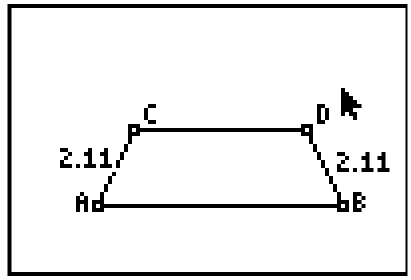


Construct a perpendicular to $\overline{AB}$ through point *Q* and a line through *C* that is parallel to $\overline{AB}$. Construct point *D* at the intersection of these two lines.



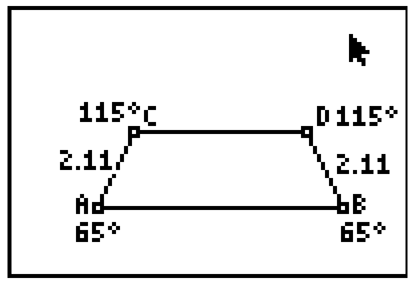
Hide the parallel and perpendicular lines and points *M*, *P* and *Q*. Complete the trapezoid and measure $\overline{AC}$ and $\overline{BD}$.

- Explain why these line segments are congruent.



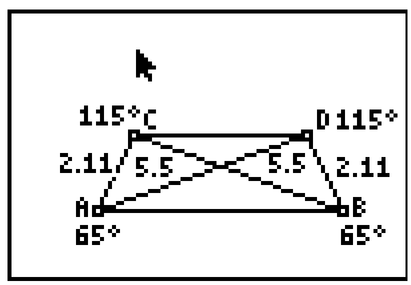
Measure the base angles—the interior angles at *A*, *B*, *C* and *D*.

- Which angles are congruent? Did you expect those angles to be congruent? Can you prove that they are?



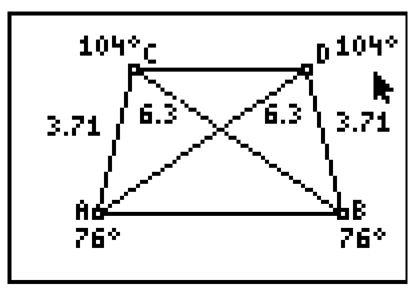
Construct and measure the lengths of the two diagonals $\overline{AD}$ and $\overline{BC}$.

- Should they be congruent? Can you prove that they are?

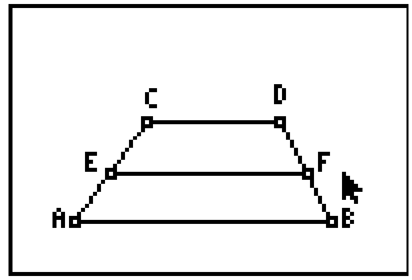


Drag point *C* and watch the angles and sides.

- Are all of the properties established above preserved?

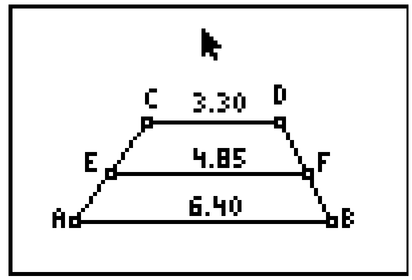


However, one property that is common to all trapezoids is that the line segment connecting the non-parallel sides is also parallel to these sides and its length is half the sum of the parallel sides. Construct a trapezoid *ABCD*. Construct the midpoints at *E* and *F* and construct $\overline{EF}$.

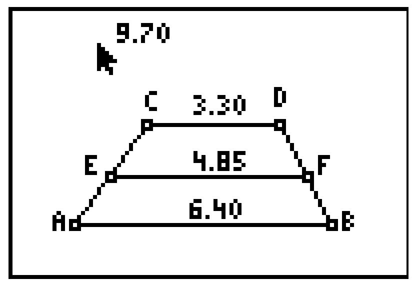


Measure the lengths of $\overline{CD}$, $\overline{EF}$ and $\overline{AB}$.

- How could you prove that $\overline{EF}$ is parallel to $\overline{CD}$ and $\overline{AB}$?



Use the Calculate tool to find the sum of the lengths of $\overline{CD}$ and $\overline{AB}$.



Place the number "2" on the screen and divide the sum by 2 using the calculate tool.

- Will this result always equal the length of $\overline{EF}$? Can you prove this result?