Math Objectives

- Students will visualize the graph of a function's derivative by considering the slope of the graph of the original function.
- Students will relate increasing/decreasing behavior of the function to the sign of its derivative.
- Students will look for and make use of structure. (CCSS Mathematical Practice)
- Students will construct viable arguments and critique the reasoning of others. (CCSS Mathematical Practice)

Vocabulary

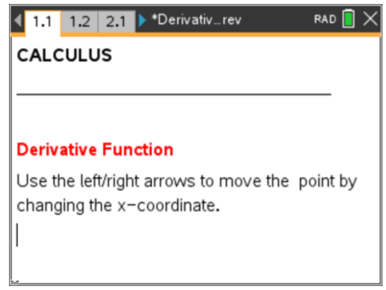
- slope of a tangent line
- derivative at a point
- derivative function

About the Lesson

- This lesson involves making the transition from thinking of the derivative at a point (i.e., as a numerical value associated with the local slope at a particular location on the graph of a function) to thinking of the derivative as a function (by considering the numerical calculation as a process that can be employed across a domain). Students will use two familiar function examples ($y = \mathbf{f}(x) = x^2$ and $y = \mathbf{f}(x) = \sin(x)$).
- As a result, students will:
    - Use a movable "zoomed-in" magnification box on the graph to consider local slope covarying with the value *x*.
    - Consider a movable tangent line with simultaneous readouts of the slope value and the *x*-value.
    - Find the formula for the derivative in the case of these two familiar examples.
    - Make the transition to thinking of a derivative as a function, opening up the opportunity to compare the behavior of the derivative over an interval to the behavior of the original function.

TI-Nspire™ Navigator™ System

- Use Screen Capture or Quick Poll to examine the values of *x* that make the expression or equation true.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point
- Click a minimized slider

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the function entry line by pressing ctrl G.

Lesson Materials:

Student Activity

- Derivative_Function_ Student.pdf
- Derivative_Function_ Student.doc

TI-Nspire document

- Derivative_Function.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

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Discussion Points and Possible Answers

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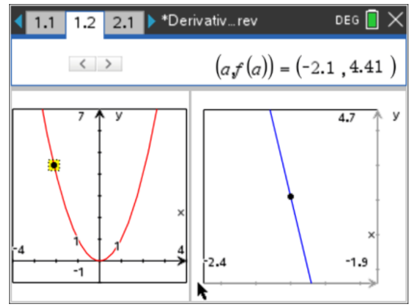

1. The graph shown on the left is $y = \mathbf{f}(x) = x^2$ with one point (a, $\mathbf{f}(a)$) boxed in. A magnified "zoomed-in" view of the box is shown on the right with the slope $\mathbf{f}'(a)$ of the tangent line to the graph at that point. In fact, the graph becomes indistinguishable from the tangent line when you zoom in close. Increase or decrease the value of a by using the up/down arrows.
   a. What is $\mathbf{f}'(2)$?

      Answer: $\mathbf{f}'(2) = 4$

   b. At what value(s) of a is the derivative $\mathbf{f}'(a) = –2$?

      Answer: $\mathbf{f}'(a) = –2$ at $a = –1$

TI-Nspire Navigator Opportunity: ***Quick Poll***
See Note 1 at the end of this lesson.

   c. Fill out the following table of values for a and $\mathbf{f}'(a)$.

      Answer:

***a* =**	–2	–1.3	–0.5	0	0.7	1.5	2.1
f'(*a*) =	–4	–2.6	–1	0	1.4	3	4.2

Teacher Tip: Students must make the connection between the (subtle) distinction in notation here:

- $\mathbf{f}'(a)$ is the slope of the tangent line at $x = a$.
- $\mathbf{f}(a)$ is the y-value of the point on the graph with x-coordinate a.
- Locally (that is, on a small interval centered at $x = a$), the graph of a differentiable function is virtually indistinguishable from the tangent line. In other words, differentiable functions are "locally linear" in their behavior.

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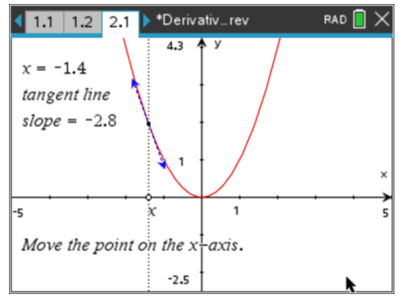

2. Grab the white point labeled *x* on the *x*-axis and move it to see the slope of the tangent line change as you move along the graph of $y = \mathbf{f}(x) = x^2$.
   a. Describe any pattern you see in the slopes of the tangent lines.

      Answer: Students may notice the slope increases or decreases at a regular rate. When the curve is decreasing, the slope is negative; when the curve is increasing, the slope is positive.

   b. Describe the relationship between each value of *x* and the slope of the tangent line at $(x, \mathbf{f}(x))$.

      Answer: Students should notice that the slope is twice the value of *x*.

> **Teacher Tip:** This is an opportunity to bring in the language of independent and dependent variables. Students are free to move the value *x* (the independent variable for the function) but are now considering the *dependent* value of the slope as it varies.

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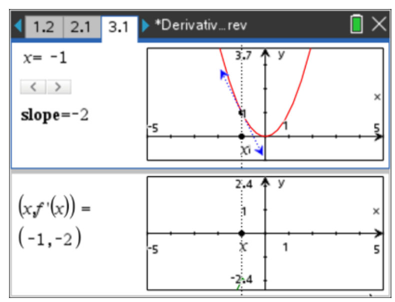

3. If you plot the value of the derivative $\mathbf{f}'(x)$ as the *y*-coordinate for each value *x*, the ordered pairs $(x, \mathbf{f}'(x))$ trace out the graph of a new function $y = \mathbf{f}'(x)$, the derivative function. Use the up arrow for *x* in the top window to see the graph of the derivative traced out.

> **Teacher Tip:** These questions connect derivative behavior to function behavior and set the stage for developing the first derivative test for extrema.

   a. What can you say about the graph of $y = \mathbf{f}(x) = x^2$ when $\mathbf{f}'(x) < 0$?

      Answer: The graph has negative slope. The *y*-values (or function values) are decreasing. The graph is going down.

   b. What can you say about the graph of $y = \mathbf{f}(x) = x^2$ when $\mathbf{f}'(x) > 0$?

      Answer: The graph has positive slope. The *y*-values (or function values) are increasing. The graph is going up.

c. What can you say about the graph of y = **f**(x) = x^2 when **f′**(x) = 0?

Answer: The slope of the graph is zero. The graph is horizontal. The graph is at its lowest point.

d. What is the equation of the graph of **f′**(x)? What is a general rule that gives a relationship between x and **f′**(x)? Explain.

Answer: **f′**(x) = 2x

> **Teacher Tip:** Many students already know the derivative of this particular function by rote memory, but this is a chance to really "see" it makes sense.

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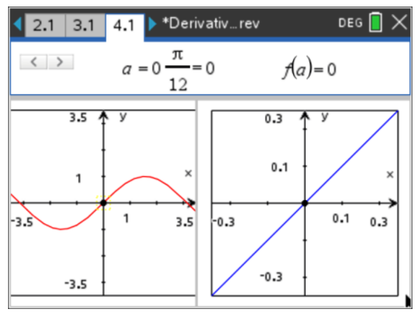

4. The graph shown in the left window is of y = **f**(x) = sin(x) with one point (a, f(a)) boxed in. Again, a magnified "zoomed-in" view of the box is shown on the right along with the slope **f′**(a) of the tangent line to the graph at that point. Increase/decrease the value of a using the up/down arrows.

   a. What is **f′**(0)?

   Answer: **f'**(0) = 1

   b. At what value(s) of a (in this window) is the derivative **f′**(a) = 0?

   Answer: **f'**(a) = 0 when $a = -\frac{\pi}{2}$ and $a = \frac{\pi}{2}$

TI-Nspire Navigator Opportunity: ***Screen Capture***
See Note 2 at the end of this lesson.

> **Teacher Tip:** Students may answer part 4b with decimal approximations. In this case, you may want to pose the question: where would the slope of the graph of y = sin(x) be *exactly* 0?

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c. Fill out the following table of values for a and $\mathbf{f}'(a)$.

Answer:

a =	$-\pi$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
$\mathbf{f}'(a)$ =	–1	0	0.707	1	0.866	0.5	0	–1

Teacher Tip: Students will not answer part 4c with exact values unless they have already concluded what the derivative function is (cos(x)).

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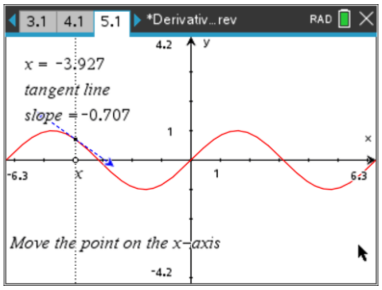

5. Grab the white point labeled x on the x-axis and move it to see the slope of the tangent line change as you move along the graph of $y = \mathbf{f}(x) = \sin(x)$.
   a. What can you say about the slope of the tangent line when the graph of $\mathbf{f}(x) = \sin(x)$ is decreasing?

   Answer: The slope of the tangent line is negative when the graph is decreasing (because the tangent line is decreasing).

   b. What can you say about the slope of the tangent line when the graph of $\mathbf{f}(x) = \sin(x)$ is increasing?

   Answer: The slope of the tangent line is positive when the graph is increasing (because the tangent line is increasing).

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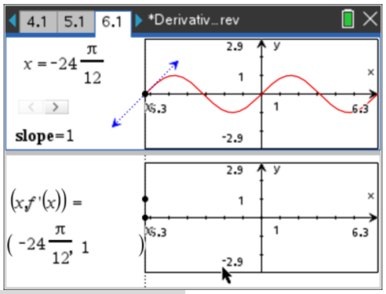

6. Use the up arrow for x in the top window to plot the graph of the derivative function $\mathbf{f}'(x)$.

Teacher Tip: These questions connect derivative behavior to function behavior and set the stage for developing the first derivative test for extrema.

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a. What can you say about the graph of $y = \mathbf{f}(x) = \sin(x)$ when $\mathbf{f'}(x) < 0$?

Answer: The graph has negative slope. The *y*-values (or function values) are decreasing. The graph is going down.

b. What can you say about the graph of $y = \mathbf{f}(x) = \sin(x)$ when $\mathbf{f'}(x) > 0$?

Answer: The graph has positive slope. The *y*-values (or function values) are increasing. The graph is going up.

c. What can you say about the graph of $y = \mathbf{f}(x) = \sin(x)$ when $\mathbf{f'}(x) = 0$?

Answer: The slope of the graph is zero. The graph is horizontal. The graph is at its lowest or highest point.

d. Does the graph $y = \mathbf{f'}(x)$ look familiar? What is the equation of the graph of $\mathbf{f'}(x)$? What is a general rule that gives a relationship between x and $\mathbf{f'}(x)$? Explain.

Answer: $y = \mathbf{f'}(x) = \cos(x)$

> **Teacher Tip:** Part 6d can be a special "aha" moment for students, even those who already know the punch line.

Wrap Up

The .tns file includes a third function (a fifth-degree polynomial) with the same sequence of screens as the two examples above (turn on the entry line to see the identity of the function). You may want to delete Problem 3 from student files and use it to assess understanding. One way to do this is to project the final screen and have students SKETCH a derivative graph, and then reveal the plotted graph.

Upon completion of the discussion, the teacher should ensure that students understand:

- How to visualize the graph of a derivative function directly from the graph of the original function.
- The relationship between the behavior of the derivative function and the original function.

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Note 1

Questions 1a and 1b, *Quick Poll*: Use an open response Quick Poll to gather students' answers to part 1a and then part 1b. If students do not all answer the same (there is only one correct value for part 1a and one correct value for part 1b), have one student be the Live Presenter and show the class how she/he found the answers.

Note 2

Question 4b, *Screen Capture*: Ask students to find one value for which the derivative is equal to 0. Use Screen Capture to take a screenshot. Some of the students will choose when $a = -1.57$ and others will choose when $a = 1.57$. Ask them to simplify the expression $-6 \cdot \frac{\pi}{12}$ and $6 \cdot \frac{\pi}{12}$, so they can connect the decimal values to $-\frac{\pi}{2}$ and $\frac{\pi}{2}$.

Assessment

Problem 3 in the TI-Nspire document provides additional opportunities for investigation or for assessment of student understanding. Comments on these additional problems are provided below.

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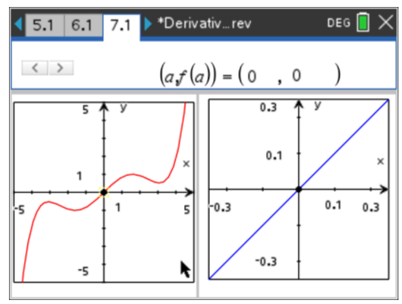

7. Increase the value of a using the up/down arrows.
   a. What is $\mathbf{f}'(0)$?

      <u>**Answer:**</u> $\mathbf{f}'(0) = 1$

   b. For how many values of a (in this window) is the derivative $\mathbf{f}'(a) = 0$?

      <u>**Answer**</u>: There are four places where the graph turns around, so there are four values of a where $\mathbf{f}'(a) = 0$.

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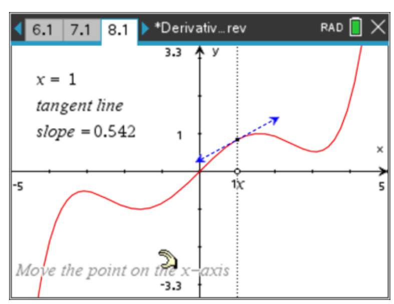

8. Grab the white point labeled x on the x-axis and move it to see the slope of the tangent line change as you move along the graph of $y = \mathbf{f}(x)$.
   a. For approximately what values of a (in this window) is the slope of the graph negative?

      <u>**Answer:**</u> The slope is negative where the graph is decreasing: $-3 < a < -1.5$ and $1.5 < a < 3$.

   b. For approximately what values of a (in this window) is the slope of the graph positive?

      <u>**Answer:**</u> The slope is positive where the graph is increasing: $a < -3$, $-1.5 < a < 1.5$, and $a > 3$.

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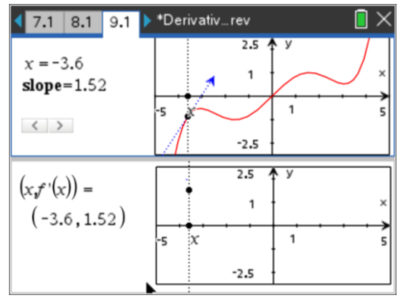

9. Use the up arrow for x in the top window to plot the graph of the derivative function $\mathbf{f}'(x)$.

   a. What can you say about the graph of $y = \mathbf{f}(x)$ when $\mathbf{f}'(x) < 0$?

      <u>Answer:</u> The graph has negative slope. The y-values (or function values) are decreasing. The graph is going down.

   b. What can you say about the graph of $y = \mathbf{f}(x)$ when $\mathbf{f}'(x) > 0$?

      <u>Answer:</u> The graph has positive slope. The y-values (or function values) are increasing. The graph is going up.

   c. What can you say about the graph of $y = \mathbf{f}(x)$ when $\mathbf{f}'(x) = 0$?

      <u>Answer:</u> The slope of the graph is zero. The graph is horizontal. The graph is at its lowest point.

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