Exploration 5

Teacher Notes

Exploration: Computing with Mathematical Formulas

Learning outcomes addressed

1.17 Evaluate any algebraic expression for real number values of its variables.

1.18 Use technology to verify the evaluation of any algebraic expression for real values of its variables.

Lesson Context

Students are sometimes introduced to algebra and algebraic operations in a formal way, using rules and procedures but without an appreciation of *why* such abstract formulations are needed. Having students evaluate formulas for given values of a variable is an excellent way to acquaint them with the idea that a variable is a "place-holder" to which various values can be assigned. A formula is then understood as an "input-output" machine that takes any value assigned to a variable, performs operations on this value, and then yields an output.

In this activity, we introduce the formula relating the time needed to cook a turkey to its mass. Even if you are conducting this activity long before Thanksgiving, this formula will be of interest to your class of hungry teenagers who will readily gobble it up. Certainly, the importance of understanding the units used in a formula will be clear from the cartoon.

Lesson Launch

Have students read the copy beside the cartoon. Ask initiating questions such as:

- Why does it take more time to cook a larger turkey?
- If we double the weight (actually mass) of a turkey, do we double the cooking time?

  answer: Yes; the function $f(x) = 15x$ is linear.
- If we change the units in the formula $f(x) = 15x$ from customary to metric units, does the formula change?

  answer: Yes. Since 1 kg ≈ 2.2 lb, we must cook a 1-kg turkey 2.2 times as long as a 1-lb turkey. The formula becomes $f2(x) = 15(2.2)x$. That is, if x is the mass in kg, then the cooking time in minutes is given by $f2(x) = 33x$.

Lesson Closure

Discuss with students the idea that formulas are procedures for computing the value of one variable corresponding to the value(s) of one or more other variables. Usually this computation will require the use of technology such as that offered by TI-*n*spire. This is particularly evident in the *TI-nspire Investigation* of this activity in which students explore exponential growth. The outputs grow slowly at first, and then they explode.

Student Work Sheet

Exploration: Computing with Mathematical Formulas

Read the discussion about the formula relating the time required to cook a turkey to the weight of the turkey. Then complete these questions.

❶. Write a formula expressing the time in minutes required to cook a turkey that is x pounds in weight. Use your formula to complete each sentence.

a) The time required to cook a turkey of 12 pounds (at 325˚F) is ______ minutes.
b) The time required to cook a turkey of 16 pounds (at 325˚F) is ______ minutes.
c) The time required to cook a turkey of 20 pounds (at 325˚F) is ______ minutes.
d) The time required to cook a turkey of 20 pounds (at 325˚F) is ______ hours.

❷. Use the calculator application of TI-*n*spire to verify your answers to ❶.

❸. To define $f(x)$ in TI-*n*spire. select: menu > **Actions** > **Define** and press these keys:

F (X) = 1 5 X enter

❹. Use the keying sequence below to evaluate $f(12)$ in TI-*n*spire.

F (1 2) enter

❺. Evaluate $f(16)$ and $f(20)$ and compare with your answers in ❶b and ❶c.

❻. A cell phone company charges \$25 per month plus \$0.02 per minute of use. Write a formula $g(x)$ that gives the monthly cost of the cell phone for x minutes of use.

$g(x)$ = ______________________

❼. Define $g(x)$ in TI-*n*spire. Evaluate $g(120)$, $g(200)$ and $g(350)$.

$g(120)$ = _________ $g(200)$ = _________ $g(350)$ = ________

❽. The formula for converting c degrees celsius into Fahrenheit degrees is given by:

$$f(c) = (9/5)c + 32$$

Define $f(c)$ in TI-*n*spire.

To construct a function table for $f(c)$ access the *Lists & Spreadsheet* application by pressing on ▸▸▸ enter. Then select menu > **Table** > **Switch to Table**.

❾. Highlight the formula f and press **enter** to obtain a table of values for $f(c)$.
Scroll up or down to find the degrees Fahrenheit corresponding to –20˚C, –5˚C and 15˚C.

TI-nspire Investigation

Follow the instructions in the TI-*n*spire Investigation in the *Exercises.*
Then complete these statements:

a) On July 31, Anita earns \$________________ .
b) The total amount earned by Anita in the month of July is \$________________.

Exploration 5: Computing with Mathematical Formulas

Formulas are recipes for calculating. They are used in mathematics, science and everyday life to determine flight paths of rockets, to cook turkeys and to compute costs of production in business. For example, the recipe for cooking the Thanksgiving turkey is simple. Put it in the oven at 325˚F until it's cooked. But for how long?

The cooking time depends on the mass of the turkey. Why? The formula $f(x)$ for cooking a turkey of mass x is given by $f(x) = 15x$, where x is the mass of the turkey in pounds and $f(x)$ is the cooking time in minutes. The gentleman in the cartoon did not pay attention to the units in the formula and incinerated the turkey.

Example 1

a) Apply the turkey-cooking formula $f(x) = 15x$ to calculate the times that are required to cook turkeys of these masses: (i) 12 pounds. (ii) 16 pounds. (iii) 20 pounds.

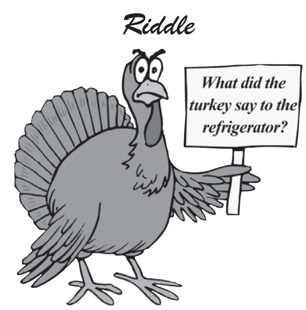
b) Enter the turkey-cooking formula into your TI-*n*spire handheld to compute the times in *part a*.

Solution

a) (i) To calculate the cooking time for a 12-pound turkey, we evaluate the formula $f(x)$ at $x = 12$. That is, we substitute 12 for x into the formula to get 15×12 or 180.
We write: $f(12) = 180$. This means that a 12-pound turkey should cook for 180 minutes or 3 hours.

(ii) Proceeding as in (i) above, we write $f(16) = 15 \times 16$ or 240. This means that a 16-pound turkey should cook for 240 minutes or 4 hours.

(iii) Proceeding as in (i) and (ii), we write $f(20) = 15 \times 20$ or 300. This means that a 20-pound turkey should cook for 300 minutes or 5 hours.



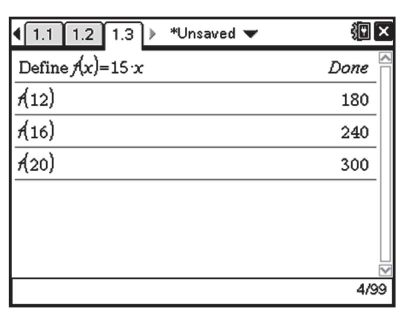
For the answer to the riddle, see *Exercise 1*.

b) To enter the cooking formula into our TI-*n*spire, we access the *Calculate* application on the *Scratchpad* by pressing (on) (enter). Then, we enter:

(menu) > **Actions** > **Define** > F (X) = 1 5 × (enter)

(i) To calculate $f(12)$, we press: F (1 2) (enter) and obtain 180 as in the display.

(ii) and (iii) We proceed as in (i) to obtain 240 and 300 in the display.



Worked Examples

A formula can be named using any letter such as $g(x)$, or a letter followed by a number such as $f3(x)$. Upper case letters for formulas are automatically changed to lower case.

Example 2

A cell phone company charges \$25 per month plus \$0.02 per minute of use.

a) Calculate the total cost of the cell phone for a month in which the minutes used are:
(i) 120 (ii) 200 (iii) 350

b) Define a formula $g(x)$ on your TI-*n*spire handheld that calculates the cost in dollars for x minutes of use.

c) Construct a table showing the values of $g(x)$ corresponding to integral values of x.

Solution

a) (i) The cost for 120 minutes is $\$25 + 120 \times \$0.02 = \$27.40$.
(ii) The cost for 200 minutes is $\$25 + 200 \times \$0.02 = \$29.00$.
(iii) The cost for 350 minutes is $\$25 + 350 \times \$0.02 = \$32.00$.

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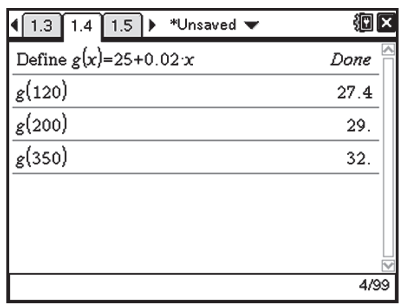
b) We first access the *Calculator* application by pressing (on) (enter). To define the formula $g(x)$ that adds to 25 the product of 0.02 and the number of minutes, x, we select: (menu) > **Actions** > **Define** and press these keys:

G (X) = 2 5 + 0 . 0 2 X (enter)

(i) To calculate $g(120)$, we enter: G (1 2 0) (enter) to obtain 27.4, indicating that the cost is \$27.40.

(ii) and (iii) We proceed as in (i) to obtain \$29.00 and \$32.00 respectively, as shown in the display.

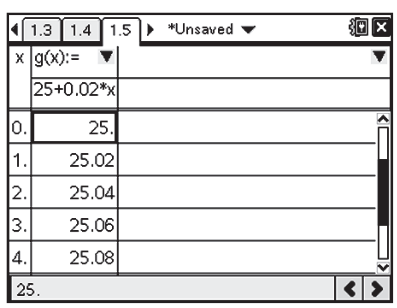


c) To obtain a function table for $g(x)$, we access the *Lists & Spreadsheet* application by pressing (on) ▶▶▶ (enter).

Then we select (menu) > **Table** > **Switch to Table**.

We then select the formula ***g*** by highlighting ***g*** and pressing (enter). We obtain the table shown in the display that gives the cost for each minute of use. We can scroll up and down to scan this table of values.

The display shows that when $g(x) = 25$, $x = 0$. *Example 3* shows how we can use a function table to find the value of x at which a formula takes a particular value.



Worked Examples

The variable in a formula is usually denoted by an x, but sometimes it's helpful to use a different letter as a variable, as in the following example.

Example 3

Most countries measure air temperature in degrees celsius, denoted by ˚C. When the temperature in Saudi Arabia is 40˚C, it is over 100˚F! The formula for converting degrees celsius to Fahrenheit degrees is:

$$\text{temperature in °F} \longrightarrow f(c) = \frac{9}{5}c + 32$$

temperature in ˚C

a) Define the formula $f(c)$ in TI-*n*spire and use it to convert 0˚C, 36˚C, and 100˚C to degrees Fahrenheit.

b) Construct a function table for $f(c)$. Use the table to convert –20˚C, –5˚C, 15˚C, and 40˚C to degrees Fahrenheit.

Use your function table to discover the temperature at which the celsius and Fahrenheit temperatures are the same number.

"LET'S GO OVER TO CELSIUS'S PLACE. I HEAR IT'S ONLY 36° OVER THERE."

Solution

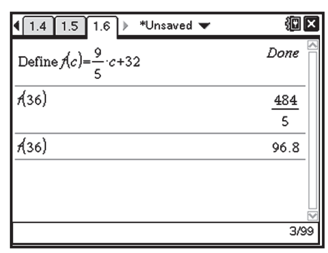
a) To define $f(c) = \frac{9}{5}c + 32$, we press these keys while in the *Calculator* application:

we select: (menu) > **Actions** > **Define** and press these keys:

F (C) = 9 ÷ 5 C + 3 2 (enter)

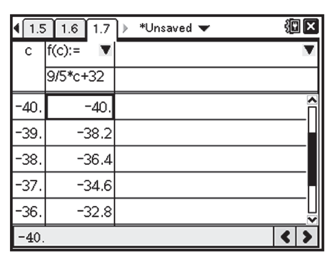
We check the screen display to verify that the formula has been defined as intended.

To compute the Fahrenheit temperatures corresponding to 0˚C, 36˚C, and 100˚C, we enter $f(0)$, to obtain 32, i.e., 0˚C is the same temperature as 32˚F, so water freezes at 0˚C. $f(100)$ yields 212, so we conclude that 100˚C is the boiling point of water.



When we enter $f(36)$, we obtain 484/5 (if in *auto* mode). To convert this to a decimal, we press (ctrl)(enter) to obtain 96.8 as shown in the display. That is, 36˚C is 96.8˚F. The gentlemen in the cartoon may want to bring some cold lemonade to Celsius's place.

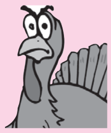
b) To obtain a function table for $f(x)$, we access the *Lists & Spreadsheet* application by pressing (on) ▸▸▸ (enter). Then we select (menu) > **Table** > **Switch to Table**.



We then select the formula ***f*** by highlighting ***f*** and pressing (enter). We obtain the table shown in the display that gives the temperature in degrees Fahrenheit for a given temperature in degrees celsius. Scrolling through the table, we find the corresponding temperatures are respectively: –4˚F, 23˚F, 59˚F, and 104˚F. Scrolling upward, we find –40˚C = –40˚F.

Exercises and Investigations

❶ Answer to the riddle on page 27.



Close the door, I'm ...

f(7/15)	f(1/3)	f(4/3)	f(4/3)	f(3/5)	f(14/15)	f(7/15)

f(4/15)	f(6/5)	f(1/3)	f(19/15)	f(19/15)	f(1/3)	f(4/15)

Calculate the cooking time above each box using $f(x) = 15x$. Then substitute the letter that occupies that position in the alphabet, e.g., 1➔A, 2➔B, and so on.

❷. Evaluate each formula for the given value of the variable.
a) $f(x) = 3x - 7$ for $x = -9$ b) $g(n) = 5(n + 4)$ for $n = 2$
c) $p(x) = x^2 - 2$ for $x = -2$ d) $q(c) = 3c^2 - 5c + 1$ for $c = -3$

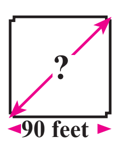
❸. Use TI-*n*spire to verify your answers in *Exercise* ❷.

❹. a) Write the formula $f(x)$ that gives the perimeter of any square with sides of length x.

b) Use your formula to find the perimeters of the squares with sides of these lengths: (i) 5 cm (ii) 8.5 m (iii) 3¼ feet.

c) What is the length of the sides of a square with perimeter 26 feet?

❺. The length of the diagonal of a square is $\sqrt{2}$ times the length of any side.
a) Write the formula $d(x)$ that gives the length of the diagonal of a square with sides of length x.



b) Use your formula to compute, to the nearest foot, the distance from home plate to second base on a major league baseball diamond. (The distance from home plate to first base is 90 feet.)

❻. a) Write the formula $A(x)$ that gives the area of any square with sides of length x.

b) Use your formula to find the areas of the squares with sides of these lengths: (i) 5 cm (ii) 8.5 m (iii) 3¼ feet.

❼. Use TI-*n*spire to verify your answers in *Exercise* ❻.

❽. A cell phone company charges \$20 per month plus \$0.03 per minute of use. Write a formula $C(t)$ for the cost in dollars for t minutes of use. Use your formula to calculate the costs corresponding to these usage times.
a) 10 minutes. b) 80 minutes. c) 4 hours.

❾. a) Using TI-*n*spire, define the cooking formula $f(x) = 15x$ as in *Example 1*. Use $f(x)$ to calculate the time required to cook a turkey of mass 24 lbs.

b) Access a function table for $f(x)$ as in *Example 3*. Use your function table to determine the mass of the turkey corresponding to each cooking time.
a) 165 minutes. b) 3½ hours. c) 5¾ hours.

❿. The height of an Asian male, measured in centimeters, is about 81.45 + 2.39 times the length of his shin bone in centimeters.
a) Write a formula $h(s)$ that yields the height h as a function of the length s of the shin bone. Use this formula to calculate the heights corresponding to shin bones of these lengths.
i) 32 cm. ii) 38 cm. iii) 0.45 m.
b) Construct a function table to determine the shin bone lengths (to the nearest cm) that correspond to these heights.
i) 160 cm. ii) 180 cm. iii) 2 m.

⓫. The surface area S of a cylinder of diameter x and height h is given by $S = \pi hx + \pi x^2/2$. Define $S(x, h)$ in TI-*n*spire and calculate the surface areas of beverage tins with these dimensions:
a) diameter 6.0 cm height 12.5 cm b) diameter 6.6 cm height 10.4 cm c) diameter 8.0 cm. height 7.1 cm.

TI-nspire Investigation

Anita has signed a strange contract with Mrs. Marples to baby sit every day from July 1 to July 31. The contract pays Anita 1¢ for the first day, 2¢ for the second day, 4¢ for the third day, and so on. The amount paid for each day is double the amount paid for the previous day.

a) Define the formula $f(n) = 2^{n-1}$ where $f(n)$ is the amount Anita is paid for her work on the nth day.

b) Construct a function table for $f(n)$ and use it to determine Anita's payment on the 20th day.

c) What is the first day that Anita earns more than \$1,000,000?

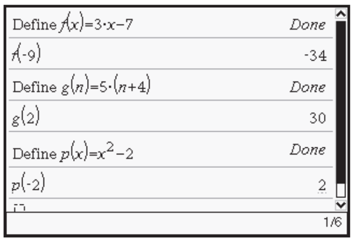
Answers to the Exercises & Hints for the Investigations

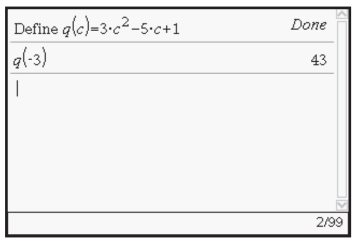
Exploration 5

❶. **Answer to the riddle:** Close the door, I'm *getting dressed.*

❷. a) ⁻34 b) 30 c) 2 d) 43

❸.



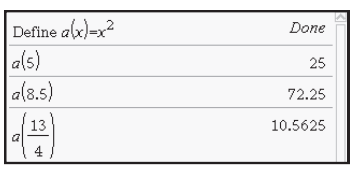


❹. a) $f(x) = 4x$
b) (i) $f(5) = 20$ cm (ii) $f(8.5) = 34$ cm (iii) $f(13/4) = 13$ feet
c) $26 \div 4$ or $6^1/_2$ feet

❺. a) $d(x) = \sqrt{2}\,x$
b) $d(90) = 90\sqrt{2} \approx 127$ feet

❻. a) $A(x) = x^2$
b) (i) $A(5) = 25$ cm² (ii) $A(8.5) = 72.25$ cm²
(iii) $A(13/4) = 10.5625$ square feet

❼.

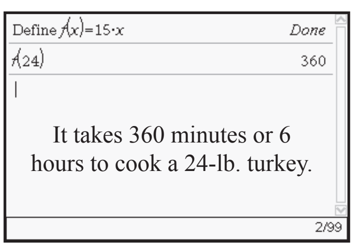


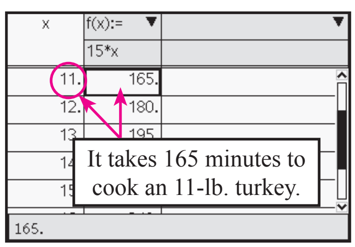
❽. The cost in dollars for usage of t minutes is given by:

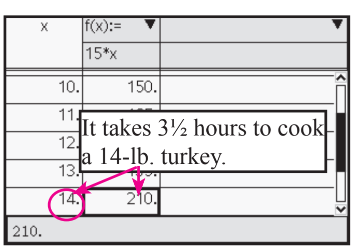
$$C(t) = 20 + 0.03t$$

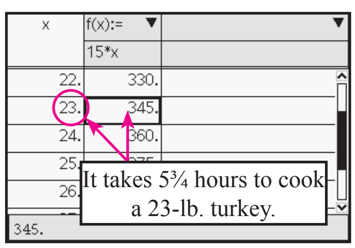
a) $C(10) = \$20.30$
b) $C(80) = \$22.40$
c) $C(240) = \$27.20$

❾.









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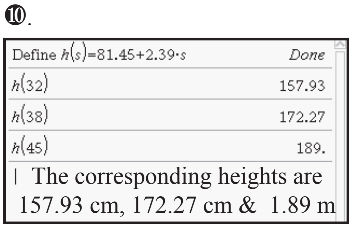

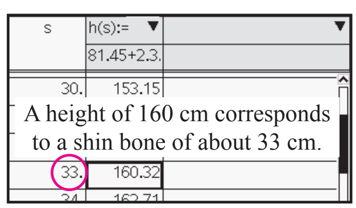

Exploration 5 *cont'd*

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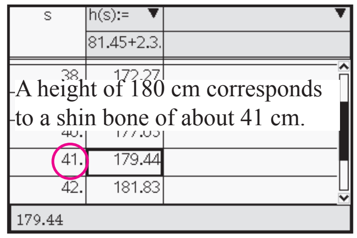

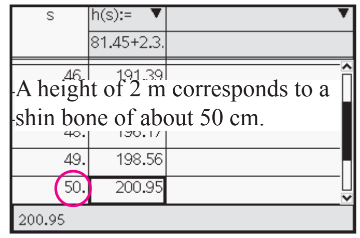

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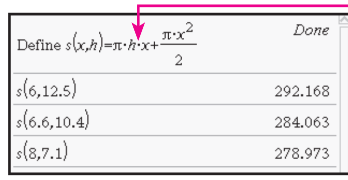

Remember to insert a × symbol between the h and the x when defining the function $S(x, h)$.

TI-nspire Investigation

The amounts that Anita earns form an interesting series. When we total the amount that Anita has earned in her first n days it is:

$$1 + 2 + 4 + 8 + \ldots + 2^{n-1}.$$

This is 1¢ less than she will earn the next day. We can check this by making a list of the amounts that Anita earns in the first 10 days beside a list of the total amounts that Anita has earned after 1 day, 2 days, 3 days, etc. Try this.

If you are familiar with spreadsheets, create these lists as adjacent columns of a spreadsheet and look for a pattern.