Change of Base

ID: 9468

Time required
40 minutes

Activity Overview

In this activity, students discover the change of base rule for logarithms by examining the ratio of two logarithmic functions with different bases. It begins with a review of the definition of a logarithmic function, as students are challenged to guess the base of two basic logarithmic functions from their graphs. The goal of applying the properties of logarithms to add these two functions is introduced as a motivator for writing them in the same base. Students explore the hypothesis that the two functions are related by a constant first by viewing a table of values, then by exploring different values for the two bases. Finally, they prove the change of base rule algebraically and apply it to find the sum of the two original functions.

Topic: Logarithmic Functions

- *Derive and use the equation $\log_a x = \log_a b \cdot \log_b x$ to convert a logarithm to another base.*

Teacher Preparation and Notes

- *Prior to beginning this activity, students should have experience applying the other properties of logarithms and solving simple*
- *This activity also offers deeper insight into the definition of a function through a discussion of what it means for two functions to be equal.*
- ***To download the DIFFBASE program and student worksheet, go to education.ti.com/exchange and enter "9468" in the keyword search box.***

Associated Materials

- *ChangeOfBase_Student.doc*
- *DIFFBASE.8xp (program)*

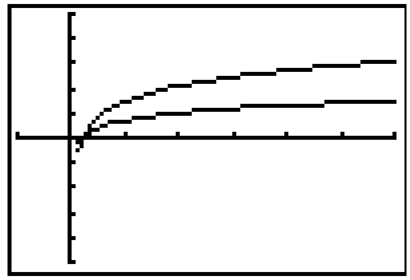
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Properties of Logarithms (TI-84 Plus family) — 9606*

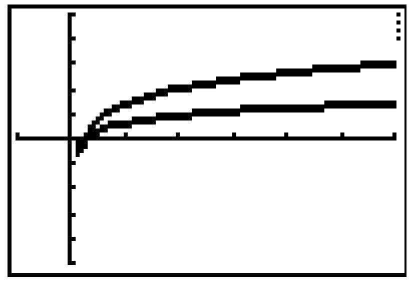
Problem 1 – Relating log functions with different bases

Students will use a program to view the graphs of two logarithmic functions. Students are told that they are of the form **Y1**(x) = $\log_a x$ and **Y2**(x) = $\log_b x$.



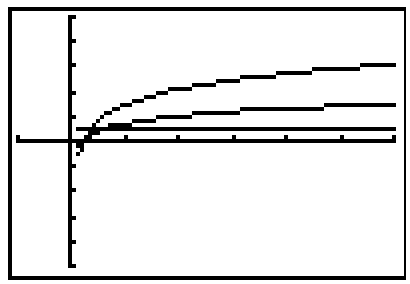
Students can use the cursor to find the coordinates of points on the graphs to approximate a and b. Students will use the program to enter their guesses to see how close they were.

The program graphs two logarithmic functions with bases guessed as thick lines on top of the original graph. If chosen a and b correctly, the graph will look like the one shown.

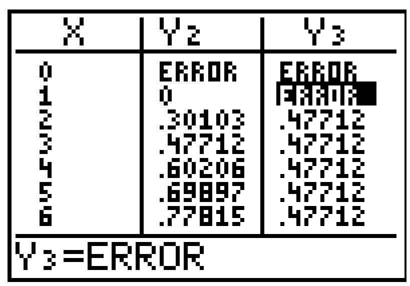


If there are more than 2 curves on the graph, try different values for a and b.

Students are presented with a motivation for wanting to change the base of a logarithmic function. Suppose these two functions were related by a constant factor. That would make things much easier. Stress the fact that we do not know that such a c exists at this point, but are following a hunch.

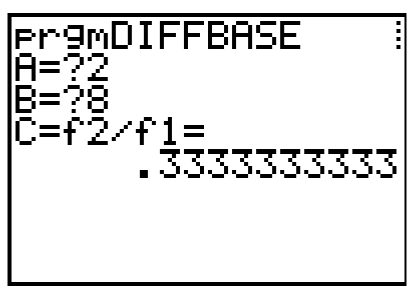


Students will use the program to find the value of c when values for a and b are supplied. The function is stored in **Y3**, so students should examine a function table of **Y1**, **Y2**, and **Y3**.

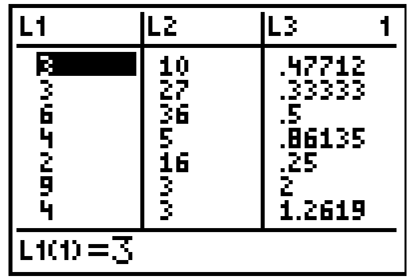


Problem 2 – A closer look at *c*

In Problem 2, students test different values of a and b (the bases of the logarithmic functions) to see how they affect the value of c. Students can work independently to enter values for a and b in the program. These values are stored in **L1**, **L2**, and **L3**. Guide students to guess a rule for c based on this data.



After students have tried at least 10 different values for *a* and *b*, exit the program and view the data in the lists. (Press [STAT][ENTER].) They should try calculating 1/*c* in **L4**.



Next, students are stepped through the process of deriving the Change of Base rule algebraically. Make sure students understand that if they can find a way to rewrite the function values at a generic point (x, y), they have rewritten the function.

Student Solutions

- a = 3 and b = 10
- The most informative points on the graphs will be those where y = 1 or some other whole number. (When y = 1, x is equal to the base of the logarithm.)
- Rewrite $\log_a x = y$ as an exponential expression.
  $a^y = x$
- We want an expression with base b logarithms, so take $\log_b$ of both sides.
  $\log_b\left(a^y\right) = \log_b x$
- Simplify using the properties of logarithms.
  $y\log_b a = \log_b x$
- Solve for y.
  $y = \dfrac{\log_b x}{\log_b a}$
- Use this formula to find $(\mathbf{Y1}+\mathbf{Y2})(x)$ if $\mathbf{Y1}(x) = \log_3(x)$ and $\mathbf{Y2}(x) = \log_5(x)$.

$$\log_3 x + \log_{10} x$$
$$= \frac{\log_{10} x}{\log_{10} 3} + \log_{10} x$$
$$= \frac{1}{\log_{10} 3}\left(\log_{10} x + \log_{10} 3 \cdot \log_{10} x\right)$$
$$= \frac{1}{\log_{10} 3}\left(\log_{10} x + \log_{10} x^{\log_{10} 3}\right)$$
$$= \frac{\log_{10} x^{1+\log_{10} 3}}{\log_{10} 3}$$