**Concepts**

The Fundamental Theorem of Calculus, Part 1, provides the connection between differential and integral calculus. If is a continuous function on , then the functiondefined by



is continuous onand differentiable on  and  In words, this theorem says that the derivative of a definite integral with respect to its upper limit is the integrand evaluated at that upper limit.

Here is some other common notation to illustrate the conclusion of the FTC, Part 1.



An interpretation of this expression is that integration and differentiation are inverse operations; what one does, the other undoes.

**Course and Exam Description Unit**

Section 6.4: The Fundamental Theorem of Calculus and Definite Integrals

**Calculator Files**

FundamentalTheorem.tns

**Using the Document**

FundamentalTheorem.tns: On page 1.2, the function  is defined in a Math Box. The default definition for  is . This expression can be changed by the user to allow for more in-depth discussions and conceptual questions concerning the Fundamental Theorem of Calculus.

The graph of  is displayed on Page 1.3. The values  and  can be changed by grabbing the corresponding point and dragging along the horizontal axis. The value  is displayed in the bottom pane.

Page 1.4 shows a graph of  and a graph of , aligned horizontally. The values  and  can be changed on the graph of  by grabbing the corresponding point and dragging along the horizontal axis. Page 1.5 shows the graph of  in the top pane, and the graph of its derivative in the bottom pane.

Problem 2 presents a special application in which the accumulation function is used to define the natural logarithm function. Pages 2.3 and 2.4 present graphical evidence to confirm the definition of the natural logarithm function and its derivative.

Page 1.1

|  |  |
| --- | --- |
|  | In Problem 1, the accumulation function  is examined graphically for a quadratic function . The user can grab and move the points on the graph representing  and |

Page 1.2

|  |  |
| --- | --- |
|  | The function  is defined in a Math Box. The default definition is shown in the figure to the left. The characteristics of this function provide an opportunity to discuss and discover the relationship between  and . However, the user can change this function to allow for other questions. There is also a brief description of the graphs on Page 1.3 here. |

Page 1.3

|  |  |
| --- | --- |
|  | The graph of the function  is shown in the top pane. The values  and  can be changed on the graph of  by grabbing the corresponding point and dragging along the horizontal axis. The shaded region in the graph is a geometric representation of the value of . The value of the accumulation function, , for the current values of  and  is shown in the bottom pane. |

Page 1.4

|  |  |
| --- | --- |
|  | This page presents a brief description of the graphs on Page 1.5. The top pane of Page 1.5 shows a graph of the function , the geometric shaded region representing the value , and the numerical value . The bottom pane shows the graph of . The values  and  can be changed in the top pane by grabbing the corresponding point and dragging along the horizontal axis. |

Page 1.5

|  |  |
| --- | --- |
|  | The graph of  is displayed in the top pane. The points representing  and  can be moved in the top pane only. The shaded region is a geometric interpretation of the value . The current value of  is shown in the top pane, and the corresponding point on the graph of  is shown in the bottom pane. The graph of  is displayed in the bottom pane; this graph updates dynamically as  or  changes. The graphs of  and  align horizontally so that the connections between the two functions are more apparent. |

Page 1.6

|  |  |
| --- | --- |
|  | This page presents a brief description of the graphs on Page 1.7. The top pane of Page 1.7 shows a graph of the accumulation function , the tangent line to the graph of  at , and the slope of the tangent line. The bottom pane shows the graph of . The point representing  can be moved in the bottom pane and the point representing  can be moved in the top pane. |

Page 1.7

|  |  |
| --- | --- |
|  | The graph of the accumulation function , the tangent line to the graph of  at , and the slope of the tangent line are displayed in the top pane. The value of  is displayed in the lower left, and the value of the slope is shown in the lower right. The point representing  can be moved in the top pane. The bottom pane shows a graph of the derivative  and the coordinates of the point on the graph of . The point representing  can be moved in the bottom pane. |

Page 2.1

|  |  |
| --- | --- |
|  | This page presents a general description of calculator Problem 2. Using the information discovered in Problem 1, we can define the natural logarithm function using an accumulation function. |

Page 2.2

|  |  |
| --- | --- |
|  | This page presents a brief description of the graphs displayed on Pages 2.3 and 2.4. These pages are similar to Pages 1.5 and 1.7, respectively, but use the function , and appropriate graphing windows. |

Page 2.3

|  |  |
| --- | --- |
|  | The graph of  is displayed in the top pane. The value of  is fixed at . The point representing  can be moved in the top pane only. The shaded region is a geometric interpretation of the value . The corresponding point on the graph of  is shown in the bottom pane. The graph of  is displayed in the bottom pane; this pane updates dynamically as  changes. The graphs of  and  align horizontally so that the connections between the two functions are more apparent. |

Page 2.4

|  |  |
| --- | --- |
|  | The graph of the accumulation function, in this case, , the tangent line to the graph of  at , and the slope of the tangent line are displayed in the top pane. The value of the slope is shown in the lower right. The point representing  can be moved in the top pane only. The bottom pane shows a graph of the derivative , or , and the coordinates of the point on the graph of . As  moves, the slope of the tangent line and the coordinates of the point are updated. |

**Suggested Applications and Extensions**

**Page 1.3**

Use the default function  and the default value of  to answer Questions 1-5. Remember that  is a function of  (for a fixed value of ). The values of  and  can be manipulated, the value of  is displayed in the bottom pane, and the shaded region in the top pane represents the accumulated net area bounded by the graph of  and the horizontal axis from  to .

1. Use geometry to estimate the value for . Then move the point representing  to  on the horizontal axis to find the exact value. Is your estimate for  an overestimate or underestimate? Why?
2. Use geometry to estimate the value for . Then move the point representing  to  on the horizontal axis to find the exact value. Explain why .
3. On what intervals is  increasing? Decreasing? What are the values of  on each of these intervals?
4. For what value(s) of  does  have a relative maximum? Relative minimum? Find  for each value of  at which  has a relative extrema. What does this suggest about the relationship between  and ?
5. Find the absolute maximum value and the absolute minimum value of  on the interval .
6. Let . Find  and explain why  even though there is more shaded area below the horizontal axis than above.
7. Let . Find the absolute maximum value and absolute minimum value of  on the interval . How do these answers compare with those in Question 5?
8. Let  and find . Let  and find . How do these two values compare? Is this result always true if the values of  and  are switched? Why or why not?

**Page 1.5**

Use the default function  and the default value of  to answer Questions 1-4. The bottom pane shows a graph of the function .

1. Use the graph of  to find the intervals on which  is increasing. Decreasing. What are the values of  on each of these intervals? How do our answers compare with those in Question 3 above?
2. Estimate the slope of the tangent line to the graph of  and the value of  at . How do these values compare?
3. Find the intervals on which the graph of  is concave down. Concave up. Estimate the  of the point inflection on the graph of . What do you notice about the graph of  at this value? Explain the behavior of  around this value.
4. Find an equation of the tangent line to the graph of  at the point with  2.
5. Grab the point representing  in the top pane and move it slowly to the right, until . Describe how the graph of  changes as  increases from  to 5. Try to use the graph of  to explain how  changes.

**Page 1.7**

The graph of  is shown in the top pane, and the graph of  is shown in the bottom pane.

1. Grab and move the point in the top pane representing . Verify that the value of  is the slope of the tangent line to the graph of  at .
2. Compare the graph of  in the bottom pane with the graph of  on page 1.5 in the top pane. What does this suggest about the relationship between  and ?
3. Grab and move the point in the bottom pane representing . Explain why the graph of  changes but the graph of  does not change.
4. Can you move the point representing  (in the bottom pane) such that the slope of the tangent line to the graph of  at  is ? Why or why not?

Pages 2.3 and 2.4

The purpose of this example is to use the Fundamental Theorem of Calculus to define the natural logarithm function.

Page 2.3

1. Use the graph of  and the definition of  to explain why the graph of  is always increasing.
2. Use the graph of  to give a geometric interpretation of the value of  and .
3. Use the definition of the accumulation function to explain why .
4. Use the graph of  to describe the concavity of .
5. Find the values  and . Explain how these two values are related in terms of accumulated area.

Page 2.4

1. Grab and move the point in the top pane representing . Verify that the value of  is the slope of the tangent line to the graph of  at .
2. Use these graphs to confirm and state the relationship between the functions  and .
3. Use these graphs and the definition of the accumulation function to explain why  for .
4. As  increases, describe the behavior of the slopes of the tangent lines to the graph of . Use the graph of  to confirm this observation.
5. Find the slope of the tangent line to the graph of  at  and at . Explain how these values are related and why.