

# Exponential Functions $f(x) = ab^x$





## **Math Objectives**

- Students will identify the characteristics of graphs of exponential functions of the form  $f(x) = ab^x$ .
- Students will describe how the parameters a and b affect the graphs of exponential functions of the form  $f(x) = ab^x$ , where  $a \ne 0$  and b > 0,  $b \ne 1$ .
- Students will use appropriate tools strategically (CCSS Mathematical Practice).
- Students will look for and express regularity in repeated reasoning (CCSS Mathematical Practice).

## Vocabulary

exponential function

### About the Lesson

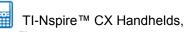
- This lesson involves moving a slider to investigate the effect of different a-values on the graph of  $f(x) = ab^x$ .
- As a result, students will:
  - Explore specific values of a and b, and then generalize the
    effects of both a and b on the graph, the domain, and the
    range of the graph of an exponential function.

# **III**I-Nspire™ Navigator™

- Use Class Capture to examine patterns that emerge.
- Use Quick Polls to check student understanding.
- Use Live Presenter to engage and focus students.
- Use TI-Nspire<sup>™</sup> Navigator<sup>™</sup> Teacher Software to review student documents.

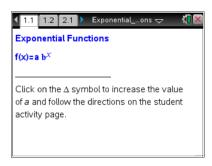
## **Activity Materials**

Compatible TI Technologies:



🌄 TI-Nspire™ Apps for iPad®, 🥌





## **Tech Tips:**

- This activity includes screen captures from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire Apps. Slight variations to these directions might be required if using other technologies besides the handheld.
- Watch for additional Tech
  Tips throughout the activity
  for the specific technology
  you are using.
- Access free tutorials at
   http://education.ti.com/calculators/pd/US/Online Learning/Tutorials

### **Lesson Files:**

Student Activity

- Exponential\_Functions\_Stu dent.pdf
- Exponential\_Functions\_Stu dent.doc

#### TI-Nspire document

Exponential Functions.tns

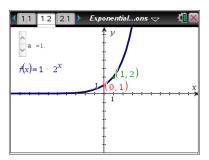
Visit <u>mathnspired.com</u> for lesson updates and tech tip videos.

## **Discussion Points and Possible Answers**

**Tech Tip:** The value of a is changed by clicking on the  $\Delta$  or  $\nabla$  symbol located in the upper-left corner of the screen.

## Move to page 1.2.

- 1. On page 1.2, you will see a graph of the function  $f(x) = ab^x$ , where a = 1 and b = 2. Click on the  $\Delta$  symbol to increase the value of a.
  - a. What happens to the graph of  $f(x) = ab^x$  as you increase the value of a?



**Answer:** The graph increases and gets steeper as you move from left to right. The graph moves closer to the *y*-axis in the first quadrant. The *y*-intercept changes from (0, 1) to (0, *a*).

b. Does the domain change?

**Answer:** The domain, the set of all real numbers, stays the same.

c. Does the range change?

**Answer:** The range, the set of all positive real numbers, also stays the same.

**Teacher Tip:** The value of b is positive. The range remained the same because you started with a positive a-value and increased it. The range will change if the value of a becomes negative. You can direct students' attention to the fact that the range does not include 0 when a is positive. On the handheld, it is hard to see that the graph is not touching the x-axis until a = 0. You might consider asking students why this is true.

- 2. Click on the  $\nabla$  symbol to change the value of a to a negative number.
  - a. What happens to the graph of  $f(x) = ab^x$ ?

<u>Answer:</u> The graph will change from an increasing to a decreasing function. Multiplying  $b^x$  (always positive) by a negative value of a always results in a negative number. Therefore, the entire graph is below the x-axis.



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b. Does the domain change?

**Answer:** The domain, the set of all real numbers, does not change.

c. Does the range change?

**Answer:** When the *a*-value is negative, the range for the function is the set of all negative real numbers.

**Teacher Tip:** Students might have noticed that when a = 0, the function no longer increases or decreases.

- 3. For the function  $f(x) = ab^x$ , find the following values.
  - a. f(0)

**Answer:** f(0) = a. Since  $b^0 = 1$ ,  $f(0) = ab^0$  or a(1).

b. y-intercept of the graph

**Answer:** (0, a) is the y-intercept.

4. Describe the graph of  $f(x) = ab^x$  when a = 0. Why does this happen?

**Answer:** The equation is f(x) = 0, which is the equation of a horizontal line (the *x*-axis). Multiplying  $b^x$  by 0 results in 0 on the right-hand side of the equation.

- 5. A function is called *exponential* if it is of the form  $f(x) = ab^x$ , provided  $a \ne 0$ , b > 0, and  $b \ne 1$ .
  - a. Why do you think that a = 0 is not included?

**Answer:** If a = 0, the function is a constant function, and its graph is the horizontal line f(x) = 0.

b. Why is the base b = 1 not included?

**Answer:** If b = 1, the function graph is the horizontal line f(x) = a.



TI-Nspire Navigator Opportunity: Class Capture

See Note 1 at the end of this lesson.



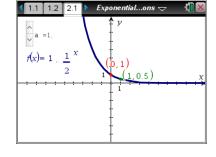
# Exponential Functions $f(x) = ab^x$

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## Move to page 2.1.

- 6. On page 2.1, you see a graph of the function  $f(x) = ab^x$ , where a = 1 and b = 1/2. Click on the  $\Delta$  symbol to increase the value of a.
  - a. What happens to the graph of  $f(x) = ab^x$  as you increase the value of a?



**Answer:** The graph falls more steeply, and the *y*-intercept shifts from (0, 1) to (0, *a*).

b. Do the domain and range change? Explain.

**Answer:** The domain, the set of all real numbers, stays the same because you can still use any real value for x. The range, the set of all positive real numbers, also stays the same because you cannot get a negative value by raising a positive base to any power.

**Teacher Tip:** The range remained the same because you started with a positive *a* value and increased it. The range will change if the value of *a* becomes negative or zero.

- 7. Click on the  $\nabla$  symbol to change the value of a to a negative number.
  - a. What happens to the graph of  $f(x) = ab^x$ ?

<u>Answer:</u> The graph will change from a decreasing to an increasing function. Multiplying  $b^x$  (always positive) by a negative number a always results in a negative number, so the entire graph is below the x-axis.

b. Does the domain change?

**Answer:** The domain, the set of all real numbers, does not change.

c. Does the range change?

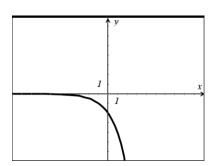
**Answer:** When the value of *a* is negative, the range for the function is the set of all negative real numbers.

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8. Look at the graph at the right of  $f(x) = ab^x$ . Margaret says that a < 0 and b > 1. Is she correct? Justify your response mathematically.

**Answer:** Yes, since  $b^x$  is always positive and the graph is below the x-axis, the value a must be negative. If b > 1 and a is positive, the graph is increasing from left to right. With a negative *a*-value, the graph is reflected about the *x*-axis.



TI-Nspire Navigator Opportunity: Class Capture

See Note 2 at the end of this lesson.

## Move to page 3.1.

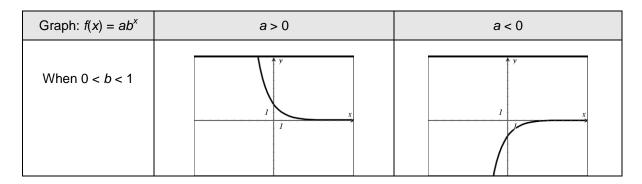
9. Sketch the graph of each of the following.

## Answer:

Graph: $f(x) = ab^x$	a > 0	a < 0
When <i>b</i> > 1		
When <i>b</i> = 1		







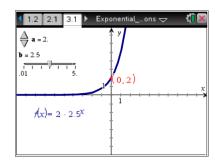
TI-Nspire Navigator Opportunity: Live Presenter See Note 3 at the end of this lesson.

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TI-Nspire Navigator Opportunity: Quick Poll

See Note 4 at the end of this lesson.

**Teacher Tip:** Page 3.1 of the TI-Nspire document can be used to explore the effects of changing both a and b in the function  $f(x) = ab^x$ . Encourage students to use this page to support their answers to question 9.



## Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- The effect of changes in the value of a on the function  $f(x) = ab^x$ .
- The effect of changes in the value of b on the function  $f(x) = ab^x$ .



## Note 1

Questions 2–5, Class Capture: Take a class capture to see how the students are progressing.

**Option:** After question 5, ask students to change the value of *a* to create a graph that is an increasing function. Discuss in groups why it is increasing. Or repeat the same activity but with a decreasing function. Use Class Capture to check students' graphs.



# Exponential Functions $f(x) = ab^x$

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### Note 2

Questions 6–8, Class Capture: Take a class capture to see how the students are progressing.

**Option:** After question 8, ask students to change the value of *a* to create a graph that is an increasing function. Discuss in groups why it is increasing. Or repeat the same activity but with a decreasing function. Use Class Capture to check students' graphs.

### Note 3

**Question 9, Live Presenter:** Use *Live Presenter* to have students illustrate the six different solutions for question 9. Also, show how there can be more than one correct answer using two different students with different correct answers on Page 3.1 of the document.

## Note 4

**Question 9, Quick Poll:** Make a quick sketch of a graph of an exponential function on the board. Also supply the following underneath the sketch (on the board):

- A. a > 0, b > 1
- B. a > 0, 0 < b < 1
- C. a < 0, b > 1
- D. a < 0, 0 < b < 1

Send the "Multiple Choice A Thru D" option to the students.

**Option:** Change the graph, and resend the same "Multiple Choice A Thru D". (You already have the A, B, C, D responses on the board.)